Workplace mathematics : a study of mathematics in use in the UK assurance division of an international accounting firm

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PhD thesis of Margaret M. Dawes

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Abstract

This study aims primarily to show how mathematics is actually performed in everyday work. It is set in the UK assurance division of an international accounting firm. The study's epistemological foundations are derived from Wittgenstein's views on rule governed practice.

It comprised observing ten principal participants, accountants and an administrative assistant, at work in eleven extended observations. Lave's theory of situated cognition, Hutchins' distributed cognition framework and textual analysis were mainly used to analyse data. Luhmann's system theory underpins a distinction made between mathematics of the discipline of mathematics and that used in everyday work. It also provides a framework for dealing with complexity.

A definition of *workplace mathematics* is grounded in participants' practices, following Nunes et al.'s definition of *street mathematics.*

The complex picture observed is reported through case studies of telling episodes. Participants' actions were substantially determined by the tasks undertaken and surrounding contexts, but their extant knowledge and skills were also critical to competent performance. The mathematics used was embedded in the tasks/texts and, through analysis, is rendered explicit. Generally no mathematics beyond that studied in the higher tier GCSE mathematics was required. Nevertheless it was used with considerable sophistication.

The findings describe

- the mathematics used,
- the extensive and varied ways in which it was used, and
- some of the factors that contribute to competent/expert performance, including collaboration and teamwork, and on the job learning and innovation.

Two additional critical findings, which are derived from theoretical considerations and observation, are:

- how we come to do and to know is dependent upon teaching, coaching and induction into practice; and
- an individual's understanding, knowledge and skill in *workplace mathematics* is achieved through the practice of engaging in work.

Although the findings are situationally specific, continuities in practice across individuals and situations observed, and findings reported in other studies enable tentative suggestions to be made about functional mathematics curricula.

Contents

1 Setting the scene

1.1 Introduction

In this thesis I describe, analyse and report upon how employees working in two business units of a UK assurance division of an international accountancy firm used mathematics as they worked. It is a study from within the situated cognition paradigm.

This research project arose out of my interest in the reform of the school mathematics curriculum. I am a mathematics graduate and have always enjoyed and valued mathematics as a subject for study. Through my son's primary school education in the late '80s/early '90s and my work as the business member of the Numeracy Task Force and related activities, I became concerned that the mathematics curriculum as realised in the classroom was not of sufficient relevance to mathematics in use in work and everyday life. In England, this aspect of reform seemed in the late 1990s to be driven by the mathematics and education communities view of how skills learnt in mathematics lessons are developed and used in mathematics lessons to solve what are perceived to be everyday problems. To someone who had spent 20 years working as an accountant, the practices did not appear to reflect how things were done within many workplaces in anything like an authentic manner. I therefore considered it worthwhile to carry out research that would enable the mathematics education community to understand how mathematics is actually used in practice within particular workplaces, and also to provide material which could be used to generate mathematical practice for the classroom that would cultivate mathematical skills necessary for much ordinary work.

In this chapter, I set out my original and modified research questions, and a brief outline of my study, its methodology and lines of enquiry. I conclude by describing the structure of the rest of the thesis.

1.2 Main research questions

My original research questions were posed as follows:

"One of the aims of mathematics education is to prepare all students for work and life. It is not, and should not be, its sole or even main purpose but it should be one of its main aims. There appears to be an unwritten presumption that the current curriculum, as delivered in schools, should provide a good grounding in the mathematics that is needed for general employment. The purpose of my proposed research is to test this presumption. The questions that I propose to research are:

- What mathematics do employees .....in non-specialist employment use at work?
- What mathematics could they have used to be more effective?
- Will the current National Curriculum and A-level mathematics equip students adequately for work?

The primary purpose is to find the mathematics actually used by employees in the course of their employment. The secondary and tertiary purposes are to use the findings to consider whether or not the employees could have been more effective if they had used other mathematics and to consider whether, together with results from other published studies, mathematics education in English schools and colleges equips students adequately for work." (Source: original research proposal.)

As I began phase II, I redefined and narrowed my main research questions. They became:

- How do employees in non-specialist employment[1] use mathematics in the workplace? and
- What insight does my fieldwork and analysis and my literature review provide into the nature of the relationships between the *workplace mathematics* of my participants and mathematics education?

The main question changed from a 'what' question to a 'how' question. Answers to the 'how' question give insights into use in performance with the result that my original main question has been subsumed into the 'how' question. The question relating to effectiveness also became partially absorbed into the 'how' question. I dropped it as a specific line of enquiry, partly as a result of the need to reduce the breadth of the study and partly because it is a question that can only be answered to the satisfaction of policymakers by taking account of the detailed analysis of results from a number of different studies of *workplace mathematics.*

1.3 Outline of study

1.3.1 Methodology

To find *workplace mathematics* I chose to find out how individuals, whose work is relatively unsophisticated from a mathematics viewpoint, use mathematics as they work. My field work was carried out in two phases in the firm where I had worked for 20 years. I observed 9 accountants (ranging from a second year student to an experienced partner) and one administrative assistant as they worked. I followed the individual in phase I, while I followed the task in phase II. I sought to capture data primarily from observations of participants as they worked (rather than through interview and/or analysis of the texts produced), in order that the main evidence for analysis was contemporaneous to action, rather than reflections on past actions.

1.3.2 Lines of enquiry

The work observed, which was initially analysed using the techniques of grounded analysis, structured the lines of enquiry followed. I observed 'expert' accountants and an 'expert' administrative assistant in their daily work. The accountants/administrative assistant engaged in activities which contributed to the performance of an audit or the administration of the business unit. My findings are reported in the main body of the thesis via detailed case studies of telling episodes observed and then summarized and developed in the three concluding chapters.

[1] For meaning see discussion in section 5.1.

1.3.3 Definition of *workplace mathematics*

I also looked for, found and developed a concept and a definition for *workplace mathematics* (chapter 6). The definition is quoted here as it is necessary background to the theoretical background in chapters 2 and 3.

***Workplace mathematics** is mathematical activity carried on within the workplace as part of ordinary work practices. Two defining principles must be present for a mathematical activity to be recognised as workplace mathematics;*

- *part or all of the work practice (system) must incorporate at least one mathematical concept*[1] *which is used in accordance with the rules of mathematics; and*
- *usage occurs when a participant acts in a way that is consistent with the mathematical rules that apply to the concept/s such that either meanings are given to information, or actions and/or events in the 'real' world are facilitated.*

1.4 Structure of thesis

Chapter 2 deals with the theoretical frameworks that underpin the thesis. Chapter 3 reviews some research literature and reports relating to *workplace mathematics.* I use further literature in chapter 6 as I develop the concept and definition of *workplace mathematics.* I delay reporting on detailed findings from the literature of mathematics in use in work and everyday life until chapter 12, so I can compare and contrast my findings with some findings reported in other studies. Chapter 4 deals with my methodology, and reliability and validity. Chapter 5 reports upon 4 out of 5 observations made during phase I as they are used to ground the definition of *workplace mathematics.* Chapters 7 to 9 use case studies to illustrate in depth the mathematics used in work and the way in which it was used via the following themes:

- the organisation and presentation of financial information;
- mathematical modelling; and
- mathematics as information, warrant and rhetoric in narrative text.

In chapters 10 and 11, I report in detail on aspects of

- learning and
- innovation

that involve *workplace mathematics.* In the second half of chapter 12, I bring to the fore the use of mathematics in social intercourse and the importance of the participants' extant knowledge and skills. I also examine how cognitive effort is distributed between team members. In chapter 13, I conclude by summarizing and categorizing my findings and in chapter 14, I consider the implications of my research for functional mathematics curricula and future research.

As I was finishing this thesis in June 2007, the Qualifications and Curriculum Authority (2007) published a draft (version 0.5) of *Functional skills standards: mathematics.* My study both shows and analyses in depth how individuals use mathematics in everyday work and hence it is directly relevant to the debate about the standards and their realisation in teaching and learning.

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[1] See section 6.2 for definition

2 From a theoretical perspective: knowledge, mathematics and the practice of work

2.1 Introduction

In sections 2 to 4, I use arguments from Wittgenstein's later philosophy to develop an epistemological stance and Luhmann's systems theory to provide sociological perspectives on mathematics and complexity. Then in sections 5 and 6, I discuss why and how the situated cognition paradigm can be used to analyse the work observed. At the end of section 6, I break with the convention of confining this part of a thesis to theoretical background alone and describe my participants' working world and by juxtaposition show how it can be mapped to the organisational framework used by Hutchins (1995). In section 7, I describe the theoretical framework that I propose to use to speculate about my participants' thoughts that were not adequately manifested in action. I conclude the chapter with a brief discussion about the disparate theories used and why I chose not to use activity theory as an analytical tool.

In chapter 3, I discuss briefly some literature relating to mathematics in everyday life and the workplace, while in chapter 12, I compare and contrast my study and its findings to findings from three key texts used. In chapter 6, I develop a principled definition of *workplace mathematics* using some of the ideas discussed here, my observations and further theoretical considerations.

2.2 A social theory of mind and meaning; what it is to do and to know

Following Williams' development of Wittgenstein's later work, I set out briefly principles for a social theory of mind and meaning and hence knowledge (Williams, 2002; and Wittgenstein, 1953 and 1978).Wittgenstein's later philosophy provides an account of: how we come to know; the critical role of learning in the acquisition of knowledge; and thus how knowledge is socially constructed.

Much of Wittgenstein's later work was concerned with attacking three contemporaneous philosophical issues – referential theories of meaning, essentialist theories of understanding and the Cartesian model of consciousness. In the course of these attacks he develops views on epistemology – essentially that mind and meaning are socially constituted. Wittgenstein's views about how we come to act and know are central to my thesis in two respects. My participants were, when they came to their workplace, already imbued with innumerable bedrock practices that enable them to go about much of the business of everyday life and work without much thought – "with right but without justification" paraphrasing Wittgenstein's words (Williams, 2002, p.49; and Wittgenstein, 1953, §§ 289). This includes much of the mathematics they used as a matter of routine. In addition when my participants engaged in non-routine activities they used the bedrock practices/techniques to facilitate and support their work, and generally they warranted new knowledge using prior knowledge and techniques they had already developed. Below I discuss Wittgenstein's views on how we come to know, the nature of the warrant attaching to that knowledge, and part of the nature of the background necessary for its acquisition.

How we come to know

In Part VI of *Remarks on the Foundations of Mathematics* (1978), Wittgenstein discusses how we might come to know something, and how we engage in rule governed practices such as arithmetic. He argues that an individual comes to know something by engaging in practice within a community. In what follows I summarize part of his argument as set out in Part VI and like Wittgenstein I use a specific case to move the argument on. How does a child come to follow the rule, '5 + 4 = 9'? The proposition '5 + 4 = 9' states that it is so but does not show how it is so. A child can only come to understand that it is so through training and guided learning, or through contemplation of a process by which he is led to the rule. At first the rule may be an empirical proposition for the child, i.e., derived from experience. Over time, however, for most children the empirical proposition hardens into a mathematical rule, '5 + 4' always equals '9'. It hardens into 'it must be so'. At this point the child has adopted a concept. When a child is asked 'is 5 + 4 = 9?', then the question is already put; and the child is left to say 'yes' or 'no'. If, one the other hand, one asks how is '5 + 4 = 9', another question is being asked; an explanation, justification or proof of the proposition is being sought. When a child is asked '5 + 4', he is being asked to add '5' and '4'. It is an order. He will comply by carrying out the addition. Here this sum always brings the same rule into play, always giving rise to the same result. The answer may be assessed in different ways. In time, the child knows the answer. He has no doubt about it. He knows it must be. He also probably has one or more procedures for obtaining the answer and possibly for spotting and correcting mistakes. But he does not doubt that '5 + 4' is '9'. Once he has grasped the rule, then he is bound in what he does when he follows the rule and he is bound in his judgement about what is in accord with the rule and what is not. Here we have both the necessity and normativity of rule following, which is discussed below. Following a rule presupposes much. It presupposes custom, agreement within a culture, agreement in action, agreement in the meaning of words and agreement in judgement.

The nature of warrant for knowledge acquired through rule governed practice

Williams argues in *Wittgenstein, Mind and Meaning* (2002) that Wittgenstein holds a community view of rules and that the objectivity of rule following is essentially social. Here 'objectivity' means that rules distinguish between correct and incorrect applications, i.e., they are normative, and rules impose a constraint upon the behaviour of an individual beyond that of mere 'say so', i.e., they impose necessity. There are two dimensions for rule governed practices and actions, a practical (psychological) dimension and a justificatory or epistemic dimension; the first guides or determines the individual's behaviour and judgment and the second assesses whether or not the action or judgement is correct. Williams argues that in what she calls the Classical Account of epistemology, the rule serves both to guide an action and to justify it, resulting in the primacy of the epistemic dimension. To avoid an unsettling gulf between what we ought to do and what we do, it is important that the rule justifying the action plays some role in its enactment. Wittgenstein successfully challenges this view. He offers no theses, only criticism and diagnosis of inadequate or wrong theorizing. He suggests a fresh start to the problem by changing the question from "But how can a rule show me what I have to do at this point?" to "What has the expression of a rule – say a sign-post – got to do with my actions?" His answer is that training (acculturation) into a custom or social practice shows how we come to follow rules and that practice "is crucial to our understanding of understanding[1]" (Williams, 2002, p.168). Meaning[2] is in use and thus a social phenomenon. In being so trained the individual becomes the master of a technique. With this new approach the epistemic and practical dimensions are replaced by the normativity and necessity of rule following. The normativity of rules is grounded in community agreement over time and the necessity of following rules is grounded in forming a second nature through the process of acculturation. A rule or master-pattern emerges from within a practice and thus normativity which enables a distinction to be drawn between a correct and incorrect practice also derives from social practice. Community agreements do not constitute justification for particular judgements. What is indispensable for appropriate judgements is concord[3], not justification by or through consensus. To follow a rule blindly using Wittgenstein's metaphor is to act "with right but without justification." The logic of action being the blind following of a rule presupposes a structure and that that is provided by the harmonious interaction of a group of people. (Closely based upon parts of Williams' arguments in chapter 6 of *Wittgenstein, Mind and Meaning* (2002), which in turn is based upon argument in Wittgenstein's *Philosophical Investigations* (1953)).

The background against which normativity and expertise are developed

My argument in this subsection draws heavily upon Williams' analysis in chapters 7 and 8 of *Wittgenstein, Mind and Meaning* (2002) and is illustrated by considering some of Wittgenstein's views on mathematical proof (Wittgenstein, 1978).

Wittgenstein's views relating to rule governed behaviour provide the basis for a social conception for what it is to do and to know. Practices such as mathematical proof or the application of accounting and auditing standards rest on a bedrock of shared customs, practices and techniques acquired through engaging in social practices, including those of everyday – as do the sophisticated and complex normative structures of academic disciplines and other group practices. Normativity developed within a practice/community is a test for correctness or validity of a rule. Here I consider normativity and the conditions in which such normativity is developed. Before I develop the argument I first describe some of Wittgenstein's views about proof by way of background. He states that a mathematical proposition is a rule while a proof of the rule is "a *blue-print* for the employment of the rule" and it, that is the proof, "stands behind the rule as a picture that justifies the rule." A proof thus justifies a rule by showing how the rule is to be used. The proof as a picture justifies the rule not to show how things are, but to produce what must be.

Wittgenstein's characterization of that background places emphasis on the obvious propositions that *hold fast* and the learning through which we acquire concepts and master techniques, which in turn lead us to the logical "must" as a "lesson....drawn from the scene." In Part VI of *Remarks on the Foundations of Mathematics,* Wittgenstein opens with the following key statements on the subject of proof. "Proofs give propositions order. They organise them. A proof is a pattern of propositions. The

[1] For Wittgenstein, understanding is associated with concepts like 'grasping a rule', 'following a rule' and 'knowing it must be'.
[2] Meaning is in usage or as Wittgenstein says "Mathematics as always is measure, not things measured."
[3] 'Concord' is agreement within a community as to what is in accord with a rule and what is not.

concept of a formal test presupposes the concept of a transformation rule and hence a technique." The point that Wittgenstein is making here is that any organized set of propositions has to be subject to a set of transformation rules if it is to be understood as a proof and indeed is a proof. To avoid the philosophical problem of regress, both the rules of transformation, which are embedded within a practice of application and use, and the technique of application are part of the necessary background within which the proof is created. The foundational question here concerns the issues of application of techniques and the production of regularities. Techniques have been mastered when it is obvious what must be done to produce particular results. Thus "Techniques are .... the regularities that create the space for going on in the same way" (Williams, 2002, p.209). Mastery of technique, as developed and judged through training within or allied to a practice, constitutes what it is to go on in the same way. Applying the rules of transformation is a skilled activity within the practice of mathematics, not a set of rules. Implicit in this is that much that *holds fast* (i.e., are bedrock practices) for the practitioner is necessary for the exercise of the skill. Two important aspects flow from Wittgenstein's concept of technique: "only through technique can we grasp a regularity" and "the technique is external to the pattern of the proof". Normative similarity arises against the background of technique; to understand a proof one has to have a technique that enables one to follow/understand the transformation rules.

Thus for Wittgenstein the propositions of mathematics are normative rather than metaphysical. He also applies similar arguments to grammatical propositions, propositions that *hold fast* (those that are obvious such as "this is a hand") and the adoption of concepts. They are not truths but norms. For Wittgenstein an object cannot be inherently a norm and the individual mind cannot be the source of a norm. So he has to characterize normativity without attempting to explain "the determination of a concept". The dimensions of a norm are the logical space of concepts and the prescriptive role of concepts. The logical space of concepts is realised by the normative behaviour of the participants within particular practices or within the relevant language game to use a term commonly used by Wittgenstein. Through practice participants come to see that things "must be like that". On the other hand the prescriptive role of propositions in participants' lives is a function of the acquisition of the relevant background techniques and their associated meanings which are developed through initiate training. These together provide an epistemological account of how expertise and expert knowledge are created socially within specialised disciplines, professions and crafts, and working environments.

These ideas also highlight the central role that learning in the community plays in induction into a practice. Learning through practice leads to the understanding of and assimilation of a practice's rules, and as a result following the rules becomes second nature or striving to follow them correctly becomes the normal procedure. A consequence of this is that transgression is the exception – this can either be due to imperfect performance or be deliberate.

My participants engaged in many rule governed practices. Those of everyday life – including those of arithmetic – tended to be blindly followed. On the other hand not all had become second nature though the normativity of much of professional/work practice was accepted and my participants sought to achieve common understandings. Wittgenstein's ideas provide several gains for those studying cognition *in situ*:

- it provides an account of how we come to understand what must be and a means of assessing the correctness or otherwise of actions and judgements, or in other words what it is to know something;
- it locates all action and judgements against a background of social practice;
- it provides an account of novice and master performance across different contexts/practices, e.g., everyday life, mathematics, and different types of work, such as accounting and auditing; and
- in my view it can also be used to provide a plausible account of how the same/similar rules may be followed in different circumstances/contexts, thus possibly resolving a common objection to the situated cognition paradigm.

Within our highly differentiated and fluid society, practices and norms developed in particular communities can and are imported into other communities and practices, and in the process may or may not be transformed.

2.3 The practice of mathematics redefined; using Luhmann's concept of society

2.3.1 Mathematics as part of workplace practices

What it is to do mathematics is contested. From the perspective of the academy, there is mathematics as perceived and practised by mathematicians and there is mathematics used elsewhere, the usage of which ought perhaps to conform with that in the academy. From the perspective of policymakers, Government and the business community, there is a demand that ideally all should receive a good mathematics education, whatever that is, and that all should be numerate. Today, being numerate is often associated with numeracy, i.e., the mathematical knowledge and skills sufficient to enable the individual to operate competently both in the workplace and everyday life. Thus to theorize about mathematics in use in work, it is necessary to be clear about what mathematics is and what it is to use it as part of workplace practices.

2.3.2 Luhmann's concept of society explained

Systems theory as developed by Luhmann, the German social theorist, particularly his concept of society, provides a sociological perspective on the question, 'What is mathematics?' and enables one to distinguish between the discipline of mathematics and the use of mathematics outside the discipline (Luhmann, 1999 and 2000).

Luhmann's concept of society is an operatively closed system, which comprises all communication. Society/communication is essentially an intermediary. It excludes humans as both living and psychic beings and the physical environment. This does not mean they are lost to theory; they are to be found not in society but in *its* environment. Luhmann proposes that the concept of communication rather than the concept of action produces society. The concept of action requires external references; it requires reference to the environment, to socially constituted complexes such as subjects and places in space. For him, the social system is an operatively closed system consisting only of its own operations produced and reproduced by communications from communications. The theoretical decisions for a conception of society as an autopoietic[1] system (a closed self reproducing system), with the concept of communication as the operation of its reproduction, belong together and are mutually conditioning.

This concept of society has implications for the role of language. It suggests that the notion of language as a system is given up. This does not mean that the phenomenon of language loses significance. The contrary is true. Language plays a role in another key aspect of system theory – the necessity for there to be structural coupling. The concept of structural coupling designates how an operatively closed autopoietic system can maintain itself in its environment. Applied to communication, language provides the structural coupling between communication and consciousness; it keeps society and the individual separate. A thought can never be a communication and a communication can never be a thought. There is no operative overlap between communication (that is society) and an individual consciousness. They are two operatively closed systems. Language in the form of speech and written texts is able to couple these systems, precisely in their different manners of operation. (Drawing and other mark making have a similar role.) Language's role does not lie in the mediation of reference but in structural coupling. This is not its only achievement. It increases the irritability of consciousness through communication and the irritability of society through consciousness, respectively transforming communication into understanding and non-understanding and internal states into communication (the former may be influenced by perception). Language also isolates society from almost all environmental events of a physical or living nature with the exception of irritations via the impulses of consciousness.

Mathematics is realised through communication and is therefore part of the social system.

2.3.3 Mathematics as a social system; an operatively closed subsystem of society

Luhmann argues that social systems, such as the legal and economic systems and science and Art, have emerged from society as society has increased in complexity and differentiated. In *Art as a Social System* (2000) he argues that over time, particularly since the Renaissance, Artworks, which are forms of communication, have gradually emerged in society and now belong to a operative closed subsystem, the social system of Art. The Art system emerged with the growing complexity of society which arises as a result of evolution and differentiation. As the Art system itself was emerging it developed a method of a codification that both belongs to and defines the system. Artworks gradually emerged from ornamentation and uses in other spheres – in churches and as palace/house decoration. Artworks include poetry, plays and the novel, installations and also some film. Codification excludes some craftwork and many but not all religious icons. Artists create artworks through the process of distinguishing and marking out the form; a process that involves both perception and communication and audiences participate in Art through perception and communication. Art is not possible without a distinguished form created by the artist (including Cage's composition of silence which requires an orchestra for performance) and perception on the part of the audience. Luhmann sees the Art

[1] In systems theory, "[autopoiesis] is a far reaching conceptual cut, which transfers self-reference from the level of structural formation and structural change to that of the constitution of elements [of a system]" (Luhmann, 1995, p.22). 'Autopoietic' here therefore means 'self-generating'.

system emerging out of society in the course of 18th Century as the system itself created its own self description and became autonomous. I am not sure that Luhmann's dating of the system is entirely correct – codification of artistic conventions govern the ancient art of Chinese scroll painting, and some 17th Century baroque painting, e.g., some works commissioned by Scipio Borghese and some of Poussin's works. Arguably the Art system may have existed in earlier periods albeit on a smaller more local scale. Two other key points here are that through communication Artworks emerge from the material conditions of the environment and some of those material conditions remain and continue to be reproduced in the environment. Design, decoration and reproductions play a role outside Art. Narratives form part of soap operas and popular novels. Most craft objects and fashion belong outside art. All influenced and influence Art and are to a greater or lesser extent influenced by Art but remain outside the Art system.

Mathematics has a similar history, though perhaps it emerged much earlier as a limited social system with geometry in Ancient Greece. From early records (e.g., Linear B tablets) it can be seen that mathematical practices were embedded in activities. With the codification of geometry (and other mathematics) by Euclid mathematical practices, ideas, conjectures and proofs were set out in text for future readers. The practices of arithmetic, mensuration and geometry continued to emerge and develop in society prior to the emergence of mathematics as a discipline. These practices developed and grew in complexity and intellectual sophistication but still in the early modern period were not clearly distinguished as a separate discipline. When Pacioli published his *Summa de Arithmetic, Geometria, Proportioni et Proportionalita* in 15th Century he divided his material as follows:

"1. Arithmetic and algebra
2. Their use in trade reckoning
3. Book-keeping
4. Money and exchange
5. Pure and especially applied geometry" (Emmett Taylor, 1956, p.179).

In the 17th Century, Newton, as a natural scientist and philosopher, created new mathematics, which in the *Principia,* is used to enable the development of his theories of physics. During Gauss's lifetime, mathematics became recognised as a rigorous discipline; its rigour providing it with the means to be self-describing. Gauss worked mainly as a mathematician. In addition to creating new mathematics he developed the rigorous methods of proof. He also applied mathematics to physics and other fields. He, himself, did not make much of the distinction between pure and applied mathematics. His practice is representative of many other mathematicians working around the end of the 18th Century. I suggest that by the late 18th Century mathematics, as understood by working mathematicians, had become an operatively closed self-describing system, within Luhmann's meaning of the term. The system is sustained both by the communication and instantiations of its practices. The dissemination of its ideas, conjectures and theories is mainly through texts (both permanent and temporary). Thus, within the concepts of Luhmann's systems theory, mathematics is a closed social system, a subsystem of society.

But mathematics/mathematical practices remained in and continue to be developed and used in the environment of the mathematics system. Included in its environment is the use of mathematics in some physics and other sciences, in the practice of statistics (as opposed to the development of statistical theory – that is developed within the discipline of mathematics), and in social science, everyday life and work. It is with this use of mathematics that most individuals engage outside education. Using Luhmann's formulation these uses of mathematics are outside the social system that is mathematics. In particular the distinction places the mathematical practices which I observed in this study outside the discipline of mathematics. This suggests that the search for *workplace mathematics* should start not with the questions 'What is mathematics?' and 'How do you recognise mathematics?' but with a different question, namely, 'How do I recognise participants using mathematics?' I answer this in chapter 6, after a review of some practices observed.

Arguably for almost all students, most of school mathematics and university mathematics (particularly that learnt for use in other subjects) is not the mathematics of the discipline of mathematics but a separate practice of the craft of mathematics for use outside the discipline.

2.4 Complexity, observation and communication

Luhmann's ideas relating complexity, observation and communication also provide a framework for dealing with the complexity of the workplace and the reflexive choices that my participants and I, as a researcher, made as we worked. This section is drawn from discussions in chapters 1 and 2 of *Niklas Luhmann's Modernity* (Rasch, 2000).

Luhmann's definition of complexity involves the notion of a quantitative threshold above which it is not possible for an observer to relate all elements of a system capable of observation. Complexity is seen as the inability to define all elements comprising a system. For Luhmann observation is an ability to make distinctions, e.g., to distinguish foreground from background or to include items (and hence to exclude items). Observation constitutes the elements of a system. Observation is not all knowing / all seeing but is selective. Selection is both construction and exclusion. Each choice precludes other choices. Thus there can be no final all encompassing overview of a complex system.

Observation, which is seen as a logical process, is a way of organising complexity. Investigation of reality includes observations on the observations of reality. Observation is also complicated by obscuring the separation between the observer and the observed. Realms of entities no longer exist as objects of study as they are reconstituted by the act of observation. Every observation is a contingent selection. An observer builds a model, writes a history or constructs a narrative in order to understand better. Such a model or narrative enables the observer to make predictions. Expectations are invariably frustrated because conditions in the system observed engender behaviour not comprehended in the model or narrative. This is deviation. Deviation is what defines complexity. Dealing with deviation is a matter of rhetoric (or political activity).

Observation is also linked with communication. Observation involves contingency and selectivity. Contingency is simply that things could be otherwise. Things are as they are as a result of selectivity. Selectivity is not only construction and exclusion, it is also the means by which information is generated. Meaning is a way of coping with complexity under the forced condition of selectivity. So observation constitutes information and meaning. Luhmann does not see communication as the process by which information is transmitted. He sees it as the synthesis of information, the communicative act and understanding by the recipient, with the individual communication event being completed by understanding. Understanding depends upon selectivity and is based upon distinguishing information from that which is communicated. Communication can be accepted or rejected. What is understood may or may not lead to further communication.

Luhmann highlights the primacy of reception in communication. A key issue is how a discrete message is transferred from sender to receiver without rendering the message unintelligible. The receiver's ability to receive the sender's intention is the ability to receive a multiplicity of intentions, not all of which were necessarily intended to be sent. There is uncertainty in all messages. The uncertainty intended by the sender can be described as desirable, and that unintended as undesirable – or as noise. If the assumption is that the unintended meaning is undesirable, the perspective taken is that of the sender. This might not coincide with that of the receiver. Luhmann considers noise to be necessary ambiguity. What makes communication both possible and necessary is the receiver's ability to differentiate between the sender's message and the selection of attitude or perspective that is a guide to the receiver's selection process. This generates new information and perpetuates communication. Too much noise hinders intelligible communication. Noise is controlled and excluded by codes as conduits of meaning. Mathematics can be seen in the world of financial reporting as the pre-eminent code for reducing noise. Once codes and intentional authority are firmly in place, noise still remains but is perceived as something other than interference (undesirable noise). It can be seen as an array of choices over and above that intended by the sender.

Luhmann's systems theory facilitates the search for *workplace mathematics* by creating a theoretical framework which deals with complexity, observation and communication (i.e., society for Luhmann). It also provides a framework for making sense of the complexity of society and its environment and how we order and give meaning to it.

2.5 The practice of work and situated cognition

2.5.1 Introduction

Solving quadratic equations using the formula in GCSE mathematics (observed in GCSE homework clubs), billing a client for cost overruns on a job (Joan – a participant), and auditing the information on directors' emoluments in a set of financial accounts (Gary – another participant) are all practices. All involve working with and creating text/texts, extracting normative meanings from the text/texts and situations, and using those meanings to progress tasks set. All to a greater or lesser extent involve using mathematics and engaging in cognitive activity. However the purpose of each task and the role of the mathematical practices within the task are different. The student solved the quadratic to practise using the formula and so learnt more about quadratic equations and their solutions. The accountant constructed a table showing the make up of the cost of the overruns with a view to negotiating an additional payment from a client. The auditor checked the mathematical integrity of the information in the draft accounts and checked its accuracy to underlying records. The context particular to the task is crucial but is not all; the execution of arithmetic operations was the same across contexts, and there were strong similarities between aspects of the social practices, e.g., all sat at a desk and all used electronic means of calculation as they worked. My thesis – an investigation of mathematics used in everyday work – is grounded in the theoretical frameworks that support the explanation and analysis of the social practice of work, of the knowledge and skills used in the work and of what it is to do *workplace mathematics.* In this section, I discuss why this study takes its primary orientation from the theories of practice and of situated cognition.

Here I discuss aspects of Bourdieu's and Giddens' theories as they provide overarching theoretical frameworks for observing and analysing work done. I also discuss the influence of Lave's theory of situated cognition on my study. In the next section, I introduce Hutchins' theory of distributed cognition, and describe how my participants' work can be analysed using a similar framework thus enabling analysis of cognition *in situ.*

2.5.2 Giddens and Bourdieu

Giddens' (1999) and Bourdieu's (1999) theories of social structure and institutional analysis provide a theoretical framework for the working world of participants and researchers.

Giddens, in formulating structuration theory, places the study of social practices at the centre of social analysis. Within the theory the term 'structure' has a specific meaning. At its most basic it refers to rules and resources. At a more sophisticated level it refers to the properties which make it possible for similar social practices to exist across varying spans of time and space. One of the main propositions of the theory is that the rules and resources that participants (in the auditor's office and in my research) draw upon in the production and replication of social action are also the means of system reproduction. Giddens also proposes that participants are very learned in the knowledge, mainly practical but also theoretical, that they possess and use in their day to day activities. My participants demonstrate this.

Bourdieu also emphasises the role of a theory of practice and in particular the concept of the *habitus.* For Bourdieu it is not sufficient to record practices, one must analyse to reveal the underlying structure. The *habitus* comprises systems of durable, transposable dispositions which generate and organise practices. To use other words, they structure structures (within which action takes place) as well as being structuring structures (partially determining the nature of action). The *habitus* is a product of history that tends to perpetuate itself by the reactivation of similarly structured practices. Participants – auditors and researchers – through their inevitable use of the *habitus* have an infinite capacity for generating different practices but this freedom is conditioned by and conditional upon the *habitus* itself.

This thesis is concerned primarily with practice rather than the permanent structures within which practice takes place, i.e., primarily with Giddens' structure and Bourdieu's *habitus*, rather than Giddens' system and Bourdieu's field of action.

Both theories emphasise continuity and regularity in practice and change brought about by environmental conditions. Both also allow for conscious intentional change, Bourdieu being more cautious:

> "[t]he *habitus* is the principle of a selective perception of the indices tending to confirm and reinforce it rather than transform it." (Bourdieu, 1999, p.117)

Structure and the *habitus* both facilitate and constrain performance and agency. My participants exercised considerable freedom in how they carried out their tasks but were constrained by their *habitus.* Although my study focuses on individual action, it, in accordance with Giddens' structuration theory, is concerned with social practices ordered over space and time rather than experiences of individual actors per se or the existence of particular institutions. Both Giddens and Bourdieu, with their emphasis on the continuity and regularity of practice and the possibility of change through conscious action, provide theoretical validity for drawing conclusions and theorizing about work and *workplace mathematics* observed and the possibility of influencing better practice.

Both theories complement Wittgenstein's views about rule governed practice as the means through which one comes to know – see section 2.2 above.

2.5.3 Lave – the theory of situated cognition

Lave's (1988) theory of situated cognition, which she developed in *Cognition in practice: Mind, mathematics and culture in everyday life* (1988) was, amongst other things, developed against the background of both Bourdieu's and Giddens' theories. The AMP[1] participants, whose activities were used to ground Lave's theory, did not 'do' arithmetic in their daily life; they engaged in everyday activities such as supermarket shopping, preparing a meal or organising everyday finances. Lave emphasises that for her participants dilemmas/problems emerged from the activities and were resolved and that sometimes in the course of resolving those dilemmas/problems they used arithmetic. A key tenet of the theory is that in order to assess how cognitive activity, in particular arithmetic in the AMP project, is realised in practice, it is not appropriate to isolate the performance of the arithmetic from the person acting in context to assess practical competence. Lave argues that the arithmetic activity undertaken and the competence with which it is performed is generated and structured by and through the contexts of the activity in which the person is engaged. In particular, she shows that performance and success in similar arithmetic tasks vary across persons, contexts and settings. All this tended to confirm that the situated cognition paradigm provided a suitable framework for analysing how individuals use mathematics as they work.

On reflection, I consider that Lave in her development of the theory of situated cognition in *Cognition in Practice* does not give adequate weight to two issues – the sources of continuities in practice and the role of individual cognitive activity. Lave shows that there are continuities and discontinuities in arithmetic practice, but tends to emphasise the discontinuities in arithmetic practice between school/the laboratory and everyday life. Also in my view her emphasis on the person acting does not give sufficient weight to the role of thought before action. Both emphases are understandable as one of her key objectives was to move research out of the

[1] AMP refers to Adult Math Project.

laboratory, school and the 'head'. She argues that it is necessary to focus on the person acting in activities in settings within a larger arena rather than on separating thinking from doing. She claims that cognition is distributed across the person acting, the activity and the settings. This in turn implies that thought is socially and culturally situated. My observations in phase I, e.g., Joan reconstructing the goodwill model[1] and Gary auditing the remuneration schedule[2], suggest that often thought precedes thoughtful action as tasks are executed in the social domain. Tasks, their contexts and a participant's own knowledge and skills provided structures and cues for *thought* and action.

Nevertheless the power of Lave's theory of situated cognition is that it provides a principled account of why mathematics in use has to be explored outside the laboratory and *in situ*. In *Cognition in Practice,* Lave (1988) shows that performance of apparently similar computational tasks are realised differently in different contexts and with differing degrees of success, and that success has to be assessed not only with respect to the normative standards of an academic discipline but also with respect to the task in hand. It was these findings that set and confirmed the agenda for my fieldwork, i.e., observation of participants as they worked. However I discovered in phase I that Lave's framework as developed in *Cognition in Practice* was not a particularly productive research tool primarily because the data against which Lave's theory was developed was concerned primarily with the individual acting in everyday activities so it does not take account of features specific to work practices in institutional settings. Although Lave and Wenger's (Lave and Wenger, 1991 and Wenger, 1998) concepts of communities of practice and legitimate peripheral participation provide a more suitable framework for analysing work practices, I decided, for the reasons set out below, to use Hutchins' (1995) version of situated cognition, distributed cognition, in phase II of my study as the framework for analysing *workplace mathematics* embedded in work practices.

2.6 Distributed cognition; work in an institutionalised setting

2.6.1 Introduction

Hutchins's theory of distributed cognition, another theory of "*naturally situated cognition*" (Hutchins, 1995), places emphasis on socially situated cognitive processes. It has added advantages in that it focuses attention on collaborative effort and teamwork and it accommodates work in an institutionalised setting. I adopted his framework and used it extensively in phase II of my study. In this section I outline the theory briefly and breaking with convention, describe how the framework can be applied to my participants' working world.

2.6.2 Hutchins – distributed cognition

In *Cognition in the Wild* (1995), Hutchins developed his theory of distributed cognition through, amongst other things, a detailed analysis of the work of the navigation teams of the helicopter carrier *Palau* and other US Naval vessels. His development of a theory of *naturally situated cognition*, unlike that of Lave, deals, in

[1] See section 8.5
[2] See subsection 5.2.3

particular, with the work of teams in institutional settings, the interrelationships between the setting and the environment, how tasks are organised, the team and the interactions between team members, as well as the role of individual team members.

Hutchins shows that the cognitive processes necessary to navigate a US naval helicopter carrier into its home port requires more than the combined intellectual efforts of individual members of the navigation team. Many artefacts and procedures used in navigation incorporate cognitive attributes derived from past activities. Navigation of the helicopter carrier would not have been possible without them:

- bearings are drawn as straight lines on navigation charts, because the charts are mercator projections which incorporate transformations of latitude and longitude. The charts are also stores of substantial information relevant to a ship's immediate environment, including the ship's proposed route into harbour;
- the alidade enables the pelorus operator, usually a relatively inexperienced rating, to take accurate bearings of the ship's position relative to particular landmarks. By aligning the hairline in the alidade with the target landmark a true bearing can be read off the scale using the gyrocompass scale inside the alidade;
- readings from the pelorus operator were transmitted over a telephone line to the bearing recorder who recorded them in the bearing log, thus storing information for subsequent use;
- the plotter used the hoey (a one arm protractor) and other aids to plot the bearing on the chart. He usually plotted three bearings, each from a different landmark, to fix each point, and with other information plotted the likely future course for the next couple of fixes; and
- previous practice enshrined in established rules and regulations including an agreed task sequence organised the activities of the team.

Artefacts such as navigation charts, the alidade and the bearing log are cognitive artefacts; they incorporate in their construction the results of past cognitive efforts. Hutchins argues that in the navigation system the environment, e.g., landmarks and stars, can operate in a similar way. Both can and do organise and/or simplify tasks/subtasks in hand. Critically Hutchins also argues that cognition does not happen only in the mind but that cognitive processes occur as individuals interact with artefacts and settings, e.g., plotting a bearing is a team task involving the pelorus operator, the bearing recorder and the plotter. Working out true bearings when the ship's gyroscope had ceased working was a joint activity of the bearing recorder and the plotter. Members of the team acting in co-ordination with the artefacts and the settings implemented the cognitive processes necessary for the successful execution of tasks.

Hutchins argues that cognition is a socially situated cultural process that is distributed within systems/subsystems. It is distributed between human beings, tools and resources (including the environment), and the settings within which activities take place. Within a workplace setting individuals/teams engage in activities to achieve tasks/goals. Through the coordination of individuals, the team and others, tools and resources, tasks are accomplished within settings. The cognitive processes through which the tasks are accomplished do not reside wholly within the minds of the human agents and their interactions with each other and their environment. They partially reside in the tools, resources and settings within which activity takes place. Past social, cultural and historical processes are locked into tools, resources and settings, giving them cognitive attributes.

This concept of cognition being a socially situated cultural process is a powerful tool for directing attention on the nature of work in an everyday setting in that it focuses attention on the setting in which a task is carried out, its social, cultural and historical antecedents, and, most importantly from my perspective, the cognitive processes that underlie work in action.

2.6.3 Work in assurance divisions of an international accounting firm

I now describe aspects of my participants' working world and map them against Hutchins' framework to demonstrate the match and why I decided to use his theory of distributed cognition. My accountant participants worked in particular business units and belonged to particular departments within those units. They mostly worked in teams that centred around audits. Each participant worked as a member of several different audit teams; there was very little common membership across the teams. The audit of the Bank took place wholly within the Bank premises. For the other audits I observed, the work was done partly at the clients' premises and partly in the office.

The setting and related cognitive artefacts Work was structured by the nature of the client's business and in particular its accounting records, the audit plan and last year's audit file, the firm's systems (including IT), and accounting and auditing practices, models and standards.

Organisation of the work The key work unit is the audit team. Most of the fieldwork is accomplished by individuals working alone on designated tasks – more than 75% from observation of part of a Bank audit. Through the review system the work done is assessed, amended and evaluated. An overall opinion on the audit is developed through work done during the planning phases, in the field and most importantly as the work done is reviewed (based partly on my observations). Usually the partner and the senior manager discuss the results of the audit with the client and its Audit Committee and the accounting firm formally reports on the audit in the company's statutory accounts.

The team Teamwork as such mostly comes into play through planning, task allocation and the review process. It is through team leadership, team membership and teamwork that the overall goals are organised, co-ordinated and realised. Throughout the working day members of the team talk to each other and other colleagues informally. They pass on messages, ask for and give help, talk about their work and related issues, as well as carrying on ordinary social conversations.

The individual An individual's own knowledge and skills play a significant role in the progress of the audit. An individual brings knowledge and skills to the job, and uses and acts upon knowledge specific to the audit. Although my participants were allocated specific tasks/subtasks which were specific to the audit, the participants conformed to agreed auditing and administrative practices. They all had considerably agency both as to the order in which the tasks were carried out and how they were done, as compared with say the navigation team of the *Palau* or a claims processing team in an insurance company (Wenger, 1998). This was inevitable given that the businesses audited differ, the issues critical to forming an audit opinion vary from industry to industry and business to business, and the need to surface the specific issues of concern relating to the business subject to audit.

Similarities between the organisation of the work of my participants and Hutchins' make his paradigm a useful tool to focus analysis. This is despite major differences between work structures. Three particular differences worth noting are: my participants' work centred around the creation of texts from texts and other evidence of business operations; the time and environmental constraints of navigation and auditing drive substantial different organizational and team structures; and as a matter of routine my participants had considerably more freedom in how they carried out their work than the navigators on the *Palau*. Finally Hutchins analysed the cognitive effort distributed through all aspects of the navigation system while my study is concerned primarily with individuals' thinking and acting as they work.

2.7 Cognition

2.7.1 An approach to cognitive processes

This still leaves the issue of how to describe and analyse the thoughts of individuals as they work. Hutchins provides a framework suitable for those who examine cognition in the wild – that is cognition outside the laboratory:

> "..........I have tried to move the boundary of the unit of cognitive analysis out beyond the skin of the individual. Doing this enabled me to describe the cognitive properties of culturally constructed technical and social systems. These systems are simultaneously cognitive systems in their own rights and contexts for the cognition of people who participate in them." (Hutchins, 1995, p.287)
>
> "..........With the focus on a person who is actively engaged in a culturally constructed world, let us soften the boundary of the individual and take the individual to be a very adaptive kind of system. Instead of conceiving of the relation between person and environment in terms of moving information across a boundary, let us look for processes of entrainment, coordination, and resonance among elements of a system that includes a person and the person's surroundings........
>
> ....I will attempt to partially dissolve the inside/outside boundary and provide a functional description of processes that could account for learning and thinking in ....cognitive activity [related to navigation]. ......[I]nternal representations are identified by their functional properties only. I make no commitment to proposed mental mechanisms or computational architectures with which the behaviours of the representations might be modelled. As far as I can tell, it is not possible to distinguish among competing models on the basis of evidence, and it is certainly not possible to do so on the basis of the sorts of evidence that can be collected in the wild." (Hutchins, 1995, p.288)

Generally I follow Hutchins's approach but try to resist an analysis that suggests violation of an inside/outside boundary between the individual psyche and communication. For the reasons see section 2.3 above on Luhmann's concept of society and the role of language.

Much of my participants' thinking was usually made manifest through their actions including conversation and the marking or creation of texts, and via the context within which they were working. However on a few occasions where this is not so, I speculate about participants' thinking and reasoning. To support these speculations, I have chosen to use schemas and cognitive models from the North American tradition of cognitive anthropology and linguistics, together with deductions from observed contextual clues.

2.7.2 Mental models; a concept from cognitive anthropology

Schemas and cognitive models are theoretical constructs used to explain mental processes, which despite their explanatory power, have been difficult to simulate. The brief discussion below is drawn from D'Andrade (1995) and *Women, Fire and Dangerous Things* (Lakoff, 1987).

D'Andrade (1995, p179) defines a schema as

> "the organisation of cognitive elements into abstract mental objects *capable of being held in working memory*[1] with default values or open slots which can be variously filled in with appropriate specifics."

It is a mental construct which enables a subject to order/structure their thoughts – thoughts about current perception, thoughts about thoughts and thoughts about action. Mandler (1984, pp.55-56) describes schemas as "organising experience". He says:

> "...activation of part of a schema implies activation of the whole, distinct from other structures and other schemas. Schemas are built up in the course of interaction with the environment.
>
> The schema that is developed as a result of prior experiences with a particular kind of event is not a carbon copy of that event: schemas are abstract representations of environmental regularities. We comprehend events in terms of the schemas they activate.
>
> Schemas are also processing mechanisms; they are active in selecting evidence, in parsing the data provided by our environment, and in providing appropriate general or specific hypotheses. Most, if not all, of the activation processes occur automatically and without awareness on the part of the perceiver – co interpreter."

The idea of a cognitive model was introduced in the 1940s by Craik (1943). Using his formulation, it consists of an interrelated set of elements which fit together to represent something. Typically one uses a model to reason with or calculate from by mentally manipulating parts of the model to solve a problem. Every schema can act as a model. But many models are not schemas themselves, but are composed of a number of schemas. My main objection to Craik's formulation is on the representative nature of mental models; my participants used models for many other purposes. I consider that Lakoff's construct of cognitive models, namely idealised cognitive models (ICMs) is a more suitable explanatory tool.

Lakoff (1987) questions the traditional view that reason is abstract and **disembodied.** He argues that reason has a bodily basis. This 'new' view encompasses imaginative aspects of reason – metaphor, metonymy and mental imagery as central to reason. He also argues that meaning is what is meaningful to thinking functioning beings and that categories are fundamental to human reasoning. He challenges the view that reasoning can be characterized merely in terms of manipulation of disembodied abstract

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[1] My emphasis

symbols. His approach is that human categorization is essentially a matter of both human experience and imagination, that is, of perception, motor activity and culture on the one hand and of metaphor, metonymy and mental imagery on the other. Consequently human reason depends upon the same factors. The way we categorise radically affects the way we reason. Lakoff suggests that evidence does not support the classical theory of categories, i.e., very simplistically, that the conceptual categories of the mind mirror the structure of the categories in the world. Instead it supports a shift to prototype based categories. A prototype is a mental model of a typical example of a category. Rosch's later view is

> "[t]o speak of a prototype at all is simply a convenient grammatical fiction; what is really referred to are judgements of degree of prototypicality" (Rosch 1978, p.40).

Lakoff accepts Rosch's point (Rosch, 1975) that prototype effects underdetermine categories and that something more is needed. He proposes that we use idealised cognitive models or ICMs. He states that we use ICMs when trying to understand the world – in particular when we theorize about the world.

But what are ICMs? They are mental models that individuals use to reason about the world. They can and do include conceptual categories. They are both idealised and cognitive. They are idealised in that they are a conventionalised way of comprehending experience in an over simplified manner. They may fit experience or they may not. The models are cognitive in that they are of the mind. It is commonplace for ICMs to be inconsistent with each other. To deal with apparent contradiction in concepts (embedded within different ICMs) account has to be taken of relevant context, mental and environmental, as this can change the meaning of the concept. The theory is concerned with conceptual structure. But structure alone does not make for meaningfulness. Additionally an account is needed of what makes structures meaningful. Conceptual structure exists and is understood because pre-conceptual structures exist and are understood. Conceptual structures are derived in part from pre-conceptual structure. Lakoff and Johnson (Lakoff, 1987, pp.266-268) propose that there are two kinds of pre-conceptual structures:

- basic level structures, i.e., basic level categories which are defined through convergence of gestalt perception, bodily movements and the ability to form rich mental structures; and
- kinesthetic image-schematic structure, comprising amongst other things; containers, paths, links, forces, balance, up-down, front-back, part-whole, and centre-periphery.

These structures are directly meaningful because they are directly and repeatedly experienced because of the nature of the body and how it functions in the environment. Abstract conceptual structure arises from basic level and image-schematic structure, e.g.,

- by metaphorical projection from the physical domain to the abstract domain, and
- through the use of basic level categories (as developed by Rosch and associates) to form super and sub ordinate categories.

Abstract conceptual structures are thus indirectly meaningful. Although the models are conceived as being mental, a key point is that knowledge of them and skill in using them is developed through the process of acculturation. Wittgenstein's later philosophy provides an account of how this might occur. Although ICMs in use are unknowable to anyone other than the subject, they are made manifest in part through what we say and do. The concept of ICMs is a powerful tool for a researcher, such as myself, to speculate about how participants theorize and reason in a field where there is some uniformity in practice.

2.8 Concluding remarks

I use many disparate theories to underpin my thesis. Each has its use. Ludwig Wittgenstein's and Meredith Williams' ideas provide me with an epistemological stance. Niklas Luhmann's systems theory provides a sociological answer to the contested question, 'What is mathematics?' It also provides me with a means to deconstruct complexity in a principled way. Selection and communication both simplify complexity, though the latter also contributes to its generation. Jean Lave's theory of situated cognition provides the justification for analysing work *in situ* while Edwin Hutchins' theory of distributed cognition provides an ideal framework for analysing cognition in institutionalised work. George Lakoff provides a principled method for speculating about my participants' thoughts.

Most of the theories were developed out of reactions, amongst other things, to referential theories of meaning, classical theories of modernity and structuralism. The exception is systems theory of Luhmann, which is a 'grand' theory of modernity, developed partially in opposition to Derrida's postmodernism and Habermas' critical theory. One of the aims of his theory is to develop a structure which can cope with the diversity of both modernity and theories about it.

There is common ground between the ideas used. All are social constructionists in their approach to knowledge. All allow a place for human agency. However all envisage that agency is constrained by custom and other practices and structures, thus enabling similar practices to be realised across different situations. Their approach to deviation is also similar. Transgression is the exception rather than the rule; it is either due to imperfect performance or deliberate.

On the theoretical front there is one further comment to make at this stage – there is little reference in this thesis to activity theory despite the fact that many recent educational studies use activity theory to provide structure for analysis. I did not use the theory as the gains to be made from using it were not obvious from my preliminary reading and analysis. My lines of enquiry and my analysis of the participants work did not fit easily into Engestrom's triangular structure (Engestrom, 1987 and Cole, 2000), although I recognise the centrality of the task perceived to be in hand in structuring what was done. In particular 'rules' (i.e., intellectual tools) as much as 'tools' seemed to mediate the subject-object relationship and there was a similar blurring between 'subject' and 'division of labour'. It is also possibly because the theory places heavy emphasis on the action generated by subject-mediator-object relationship while my emphasis was on the what and how of mediation and not on the object, i.e., the desired outcome. Hutchins' framework, on the other hand, in focusing on detailed analysis of practice in an institutionalised setting to reveal cognitive processes provides me with an organised way of looking at the activities of my participants at work such that my main research question is kept in the foreground during analysis.

3 Mathematics in the modern workplace: a review of some relevant mathematics education literature

3.1 Introduction

This brief literature review is limited mainly to consideration of aspects of how some research into mathematics use in non-specialist employment/work[1] is conducted/reported and its limitations, and overviews of literature reviews. In section 2, I critique the approach taken in the seminal report, *Mathematics Counts* (1982), and in section 3, I discuss *Mathematical Skills in the Workplace* (2002) and the CBI report, *Working on the Three Rs* (2006). In subsections 1 and 2 of section 4, I provide overviews of two recent literature reviews of mathematics in the workplace and everyday life and in subsections 3 and 4, I discuss very briefly a number of studies related to mathematics in use in the workplace and everyday life. In chapter 12, I use *Cognition in Practice* (Lave, 1988), *Street mathematics and school mathematics* (Nunes et al., 1993) and *Cognition in the Wild* (Hutchins, 1995) to compare their methodologies and some of the reported findings with those of my study. In section 5, I consider the relationship between my research questions, methods and findings and some relevant accounting and auditing research. Finally in section 6, I briefly refer to the dichotomy between education policymakers' and advisers' concerns and those of researchers working in the field using ethnographic methods.

3.2 The legacy of Cockcroft

A starting point for considering the relationship between mathematics in secondary education in England and the workplace is *Mathematics Counts,* the report of the Cockcroft Committee (1982). Its terms of reference included considering the teaching of mathematics taking into account "the mathematics required in further and higher education, employment and adult life generally." The Committee conducted a substantial review of the needs of employment, using written and oral evidence mostly from employers and trade associations, such as the CBI and the TUC, and from two research studies into the mathematical needs of various types of employment carried out by the Universities of Bath (1981) and Nottingham (unpublished). Members of the Committee also visited 26 firms. The Committee noted in the report:

> "46........ We found little real dissatisfaction amongst employers with the mathematical capabilities of those they recruit from schools except in respect of entrants to the retail trade and to engineering apprenticeships[2]....."
>
> "68. Both studies found that almost all of the mathematics which young people need to use, whatever their job, is included within all the existing O-level and CSE Mode 1* syllabuses......"
>
> "83. It is, of fundamental importance – and, we believe, not as self-evident as some might suppose – to appreciate the fact that all the mathematics which is used

[1] For definition, see section 5.1.

[2] In *Working on the Three Rs,* the CBI (2006) states that about one quarter of employers surveyed reported that they frequently encountered problems with numeracy among new non-graduate recruits.

at work is directly related to specific and often limited tasks which soon become familiar……"

The Committee perceived mathematics, amongst other things, as being a powerful means of communication, yet it did not specifically comment on whether or not the education provided through the O-level and CSE examination syllabuses provided young people with the resources to understand and use work related information which was presented in a mathematical form. The findings of the Committee with respect to mathematics and the workplace are in chapter 3 of the report and are not summarized here. Instead I would like to critique aspects of the Committee's viewpoint, as their findings have had a substantial impact on the curriculum in English schools, in particular via the report's foundation list of mathematical topics and its influence on the National Curriculum.

The Committee's focus was upon the mathematics that a young person who entered employment straight from school might use – no particular consideration was given to what the employee's needs might be throughout his/her working life. The report placed emphasis on mathematical operations used in the workplace rather than on how and why they were used. The mechanism for this was the MIST list created by the Bath team; this specifically decontextualized the mathematical operations found embedded within specific work practices. Dowling (1991) discusses how the Bath (1981) and Nottingham (unpublished) reports themselves make many references to the disjunction between school mathematics and working life. The Cockcroft report does refer to this. However, Dowling suggests that the work of the researchers, possibly contrary to their intentions, was used by the Cockcroft Committee to support the notion that school mathematics and work practices are "strongly classified with respect to each other" (Dowling, 1991, p.102). Harris and Evans (1991) consider that some of the conclusions of Cockcroft have been overstated and over-generalized with negative effects on curriculum and school/work debates.

Harris (1991) in her review of the *London into Work Project* – a study of the work of 1000 young people in inner London – comments that the frequency tables illustrating the mathematical skills used, which were developed as the project progressed, say very little about what any individual did at work. On the other hand the information about the use of mathematics embedded in the context data of the Communication Skills questionnaire showed the differences between mathematics at work and school, and recorded the way in which an individual reacted, thought and behaved. Harris comments that one of the conclusions she derived from the formal research stage of the project was that those in positions of influence continued to regard mathematics in work as a set of minimal arithmetic skills, without regard to the context within which they were set. It is arguable that in England the relationship between mathematics, education and work outlined in Cockcroft has sustained competency-based mathematics education and training, variants of which are discussed in Matthews (1991) and FitzSimons (2000).

I have other criticisms of the mathematics/work debate as set out in the Cockcroft report. The report did not consider the needs of those who entered employment with a further or higher education qualification in arts and humanities. It recognised that the early eighties were a period of rapid technological and social change but it failed to consider the potential impact of the expansion of the service sector and the decline of heavy industry and manufacturing on the nature of employment. It misread the likely impact of the advent of the silicon chip, "[r]elatively few school leavers are likely to work directly with a computer" (Cockcroft, 1982, p.40, paragraph 144). There was little or no discussion of whether or not O-level and CSE, grade 1 Mathematics and English were and would increasingly become 'gatekeepers' for employment. Arguably as a consequence, the report underestimated the importance in the last quarter of the twentieth century of mathematics to the world of work.

3.3 Recent reports relating to Mathematics in the workplace

3.3.1 *Mathematical Skills in the Workplace*

Mathematical Skills in the Workplace, the final report to the Science, Technology and Mathematics Council (Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002), sought to determine the mathematics knowledge and skills needed for employment in seven different sectors of the economy. The key finding is "that 'mathematical literacy' is displacing basic numeracy as the minimum competency required in a large and growing number of jobs", where the term mathematical literacy is defined as "the application of a range of mathematical concepts integrated with a detailed understanding of the particular workplace context." The report brings out that the workplace is pervaded by a ubiquitous but often hidden use of mathematics. It also refers to the ubiquitous use of information technology and reports that "[this] has changed *the nature of the mathematical skills required, while not reducing the need for mathematics.*" The subsequent analysis discusses in very general terms how IT and mathematical skills and practices are interrelated but only hints at the mathematics use that is incorporated within the computer systems including the output reports and the role of employees in creating, using and modifying the systems.

I have a major criticism of the report, which must cast some of its conclusions into doubt. I have concerns about whether the research methodology is capable of providing an adequate picture of the skills in use. The report states that it is not easy for companies (bodies corporate!) and their directors and employees to identify the precise role of mathematical skills in their work practices. If this is the case then research based upon interviews with managers and brief site visits focusing on mathematics use, without a thorough understanding of the businesses and their work practices, is a flawed method of determining the mathematics qualifications and skills needed as is illustrated by the following brief critique of one of the case studies.

The case study of the medium sized building society failed to bring out how the front office sales operations, which are the main focus of the case study, are related to and wholly dependent upon a large back office. The branch network in early 2003 then employed about 300 staff while the other departments, most of which are essential to the retail operations of the society, employed about 400 – some of whom were directly engaged in front office sales activities. The society employed (and actively recruits) specialists for the back office, e.g., qualified accountants and computer specialists. In particular, in February 2003 10-12 'mathematicians', mostly working some of the time as statisticians, were involved in assessing the society's risk assessment position: this included amongst other things extensive factor analysis. The 2002 published accounts of the society also reported extensively on its use of financial instruments to hedge its operations; these hedging transactions require expertise of the board and the senior management team and others in the treasury, banking, risk management and accounting departments. (Sources: the chief executive of the building society and the 2002 accounts.) The report in stating that "[o]nly very few jobs require *formal* mathematics at above intermediate level," and "given the tradition.....of promotion from within, this implies a demand of expertise [GCSE C] for more than 40% of employees," may fairly reflect the requirements of the branch operations. It is, however, not appropriate to generalise those findings across the business let alone the sector, without first understanding how the business of the society and the sector work. In the case of the medium sized building society it seems that the report may substantially under report the mathematical skills in use.

3.3.2 *Working on the three Rs*

The recent CBI report, *Working on the three Rs* (2006), explains what employers mean by functional skills[1] – namely, having the ability to tackle practical everyday tasks in the real world. It is concerned that the new GCSE functional skills modules in mathematics (amongst other things) should help to deliver sufficient mastery of functional/basic skills "that people at work require [i.e., by implication, all students will in due course require], day in and day out" (CBI, 2006, p.4). Functional skill in numeracy/ mathematics is perceived of as going "beyond manipulation of numbers to cover all aspects of mathematical awareness....." (CBI, 2006, p.5).

The CBI's findings are based upon a survey of employers and a series of 19 case studies. The case studies were all conducted in large to medium sized businesses. They were primarily based upon interviews and discussions with the HR personnel (and senior management) and focus on recruitment, training and development, perceived skills gaps of employees and describing some of the basic/functional skills needed in work. Although the studies rarely refer to actual work practices, they provide substantial useful information about functional skills used at work, and actual or perceived skills gaps, albeit that some reporting is anecdotal.

Numbers 7 to 13 of the key conclusions (appendix 1) list the mathematical skills that employers would like the functional mathematics curriculum to develop. There has clearly been an effort to keep the employers' wish list to a relatively small number of items. The list itemises knowledge and skills which are perceived to be necessary. It also places considerable emphasis on the ability to be able to calculate, understand and use/interpret mathematics embedded in information used at work and on the ability to use mathematics at work, though there is less emphasis on the latter. Point 14 of the overview and the key analysis (CBI, 2006, p.13) is particularly important. It refers to boosting awareness of the potential applications of functional mathematics in real and different contexts, accepting that this is a challenge for schools, which is already recognised. From other comments in the same section it is clear that the writers do not fully appreciate the extent of the challenge. They write that it is important to see practical uses for a skill and to practice, as this will provide motivation and reinforcement of learning, but they do not write of practice as being essential to learning itself. (Refer to section 2.2 on "what it is to do and to know".) They also write of the importance of appreciating that skills applied to one task can be applied to

[1] Throughout this report I use this definition to characterise skill in functional mathematics and other basic skills.

another without acknowledging the problematic nature of transfer (subsections 3.4.2 and 3.4.3; sections 12.2.2; and Ceci, Rosenblum and Debruyn, 1998). Finally the report makes no mention of the need to acquire knowledge and competence in the various electronic means of calculation that have replaced log tables, slide rules and some paper and pencil procedures.

The report also provides a useful summary of findings of recent basic skills surveys carried out in the last 15 years. The main overall conclusions with respect to the use of mathematics is that approximately one quarter of the adult working population's skills are to a greater or lesser extent inadequate and that low basic skills have an deleterious effect on an individual's employability. The results of the surveys are reported in a measured and fair manner.

3.4 Mathematics in the modern workplace

In subsections 1 and 2, there are two brief overviews of the literature reviews of FitzSimons (2000) and Coben et al. (2003). Subsection 3 deals with studies in the situated cognition paradigm and subsection 4 with the Techno-mathematical Literacies project.

3.4.1 From FitzSimons (2000)

Over the last 30 years there have been many reports about the mathematics skills required in the workplace. There is widespread belief that mathematics underpins and enables the structures and processes of industry. I have already discussed the Cockcroft report briefly. Foyster (1988, 1990) has observed that different mathematics skills are required at different stages of an individual's career and that what an individual is required to do depends upon his/her range of duties and responsibilities which could vary with the size of the enterprise – I prefer team. The use of information technology has become commonplace within the workplace. These comments reinforce some of my criticisms of the Cockcroft and the CBI reports. A New Zealand study by Knight, Arnold, Carter, Kelly and Thornley (1992) found evidence of the use of generic problem solving skills including the need to determine the best strategy for a task. FitzSimons comments that the research reports on mathematics in the workplace sustain the view that mathematics forms an integral and important role in activities, albeit context bound, but treats the workplace itself as being unproblematic.

The early research reports appear to be based upon the perception of a relatively static workplace – see also my criticism in section 3.2 above. More recent research reflects and recognises the changing workplace and the need for higher order thinking skills. Buckingham (1997) reports that numeracy is perceived as including the capacity to use the techniques of mathematical modelling within a production environment, and also emphasises the importance and enduring power of workplace discourses to mediate and drive workplace processes. Noss (1997) argues that within the workplace sophisticated mathematical skills are required for the interpretation of results, error detection and rectification when there is system breakdown; there is a need in many work situations to think in a mathematical way. Pozzi, Noss and Hoyles (1998) found that, with nurses and bankers, there is a need to create models in non-routine situations and that the mathematics required to deal with contested or problematic situations is far broader than basic numeracy. Noss also reports, as a result of the use of computers, that less reliance is placed upon traditional school mathematics within the workplace and that the mathematics is less visible and hence appears "both insignificant in quantity and trivial in quality." The research with bank workers revealed a split of activities between and within the front, middle and back offices of the bank such that the many employees did not, and did not need to, understand how their work fitted into context within the banking business. From all this it may be concluded that today individuals as they work need different mathematics, rather than less, as compared to twenty five years ago.

3.4.2 From Coben et al. (2003) – *Numeracy in Context (pp. 38-53)*

The chapter, *Numeracy in Context*, in *Adult numeracy: review of research and related literature* (Coben et al., 2003) covers research into mathematics in everyday life and work across four headings; Context and transfer, Investigating the use of mathematics in everyday life, Mathematics and employment, and Financial literacy.

In the section on Context and transfer, Coben et al. report that much of the mathematics in adults' lives goes unrecognised as such (Coben, 2000; Harris, 1991 and 2000; and Noss, 1997). She also discusses at some length the issue of learning and context and the possibility (or difficulty) of transfer of learning from one context to another as it has evolved from findings arising in the situated cognition and related paradigms. She does not cite literature from cognitive psychology, which also concludes that transfer is problematic (Ceci, Rosenblum, and Debruyn, 1998). The failure to achieve transfer between situations reported by both surveys (Coben et al., 2003; Ceci, Rosenblum, and Debruyn, 1998) suggests that alternative explanations for continuity of practice/problem solving across situations/practices is required.

In the section on mathematics in everyday life, Colwell (Coben et al., 2003, pp.40-46) covers the work of Lave, Nunes et al. and others working in the situated cognition/ethno-mathematics traditions. She reports on Saxe's (1991) study of children selling candy on the streets of Recife, Brazil. In particular she reports that Saxe was able to fit the activities to the four parameters of the model that he developed in connection with his work with the Oksapmin highland people of Papua New Guinea. The four parameters, which interact with 'emergent goals' are; activity structures, prior understandings, artefacts and conventions, and social interactions. This framework is not unlike Hutchins' – see section 2.6.

In the section on Mathematics and employment, Coben et al. report that survey evidence in industrialised societies show that poor numeracy carries significant disadvantage for an individual in paid work.

The rest of the section is taken up with a commentary on studies of workplace mathematics and their implications. In particular Coben et al. (2003, pp.47-48) report on the common trends identified in the different workplaces examined in the projects that form the basis for the report, *Mathematical Skills in the Workplace* (2002, p.12 and p.4). Central to research on mathematics in employment is "the importance of context and the acknowledgement of adults' pre-existing strategic knowledges as well as alternative or limited mathematical conceptions [or misconceptions]" (FitzSimons et.al., 2003, p.121). Wedege's (2001) construct of competence as a math-containing everyday competence gives weight to motivation, extant knowledge and skills, and context. Her description of competence (Wedege, 2001, p.27 quoted in pp.48-49) is grounded in the findings of her studies using the AAMT[1] technique of work-shadowing.

In the section on financial literacy, Coben el al. report on several surveys carried out in order to assess the mathematical skills or personal financial literacy of the public or sectors of the public in the UK and elsewhere. The surveys report a low level of competence; the surveys tend to be based upon questions in school-like tests with few questions being aimed at those with the lowest competence levels (Coben et al., 2003, p.34 and p.51). This is in contrast to competence reported in respect of practice within the workplace – see subsection 3.4.3 and chapter 12. Are we as incompetent as the surveys suggest? Or do surveys overestimate incompetence because they fail to take account of how we make decisions against a background of our goals, our understanding of the background necessary to generate the dilemma and its resolution, available tools (technological and intellectual), co-operation with family, friends and other advisers, and our motivation?

3.4.3 Research within the situated cognition/communities of practice tradition

Lave in *Cognition in Practice* (1988) challenged the hegemony of school math practice over usage in everyday life and subsequently work practice (Lave & Wenger, 1991) by showing that math practices used in activities in everyday life and work are substantially determined by the context in which they arise. Lave's (1988) work in the Adult Math Project, particularly with supermarket shoppers and her analysis of other studies including Scribner's (1982) analysis of working arithmetic in a dairy, together with the work of Nunes, Schliemann and Carraher (1993) and others (with street traders, farmers, carpenters and fishermen) illustrate the effect of context on the mathematics in tasks and on competence in performance. Experienced participants showed considerable competence in carrying out their everyday work and activities, and in negotiating successfully the mathematics embedded in those activities. The studies also demonstrate discontinuities in competence between solving similar problems in school-like tests and work related situations. These studies confirm the importance of studying work *in situ* in order to establish the nature of mathematical practices actually used in work. The Cockcroft report (1982) in examining workplace practices and on the job training appeared to have pre-dated these and similar studies (FitzSimons, 2000). However, by decontextualizing the mathematics practices, albeit stressing the importance of workplace methods, it failed, as is pointed out by FitzSimons (2000), to address the issue of transfer.

These studies are important because:

- they show how mathematics is realised in everyday life and the workplace;
- they challenge the hegemony of academic thinking outside school and university; and
- they also challenge conventional assumptions about cognitive transfer of knowledge and skill between situations.

[1] AAMT means Australian Association of Mathematics Teachers.

Hutchins in *Cognition in the Wild* (1995) takes navigation as it is performed by a team on the bridge of a ship as the unit of cognitive analysis, i.e., teamwork in a particular institutionalised setting. In chapter 2, he uses Marr's (1982) framework, which was intended to apply to the cognitive processes that apply inside an individual, to analyse the operation of the navigation system, thus applying a principal metaphor of cognitive science – cognition as computation – to operation of navigation. He then compares modern western navigation (pre the extensive use of the GPS system) with navigation as practised and learnt in Micronesia showing that the considerable differences between these traditions lie at the representation/algorithmic and implementation levels. He also conducts a brief historical review of the development of modern western navigation systems. Many mathematical practices are embedded in tools that aid navigation in these systems. In the process he demonstrates the situational specificity of particular practices and also shows how some of the devices and procedures enable continuity across situations.

Some of the findings from *Cognition in Practice, Street mathematics and school mathematics* and *Cognition in the Wild* relating to *workplace mathematics* are compared in chapter 12 with each other and some of the findings from this study.

3.4.4 Techno-mathematical Literacies

In the *Mathematics Skills in the Workplace* report (2002), the authors refer to 'techno-mathematical literacy' as well as 'mathematical literacy'. Hoyles et al. (2002) argue that with the increasingly ubiquitous use of information technology across the world of work, many more have to cope with the interface of mathematics/technology in their day to day work. They also argue that the mathematics with which employees in non-specialist employment[1] have to cope has changed, essentially as a result of technology. The skills seen as necessary to deal with this interface are described as Techno-mathematical Literacies (TmL). TmL are seen as combinations of mathematical, IT and workplace specific competencies that provide the capacity to deal with models and to take decisions based on the interpretation of abstract information (Kent et al., 2005).

Currently through the Techno-mathematical Literacies in the Workplace project (2003-2007), the project team[2] have been and are investigating Techno-mathematical Literacies in particular workplaces and are seeking to develop education and training materials based upon practice in the workplace for the post-16 education and training sector. The TmL project is essentially focused on improving employees' understanding and use of mathematics at the interface between the individual and the technology with which the individual works. The TmL project set out to characterise and develop TmL needed for effective practice in workplaces that are typically seen as being highly automated. The research focuses on how 'intermediate-level employees' understand and 'communicate' the mathematical aspects derived, amongst other things, from computer inputs and outputs and statistical analyses of production and other processes. Phase 1 of the study involved carrying out ethnographic case studies in 10 companies to identify and characterise TmL that are needed to function

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[1] Non-specialist employment here is employment which requires no more than 'intermediate level' mathematics – broadly grade C at GCSE.

[2] The project team is Prof. Celia Hoyles, Prof. Richard Noss, Dr. Philip Kent, Dr. Arthur Bakker and Chand Bhinder at the Institute of Education, University of London.

effectively in each workplace. Phase 2 involves action research projects with the partner companies and is concerned with designing learning opportunities using interactive software aimed at developing the TmL identified in phase 1. This research with its focus on the interface between the individual and technology and the development of teaching materials rather than individual performance *in situ* is complementary to, rather than directly relevant to, my approach.

3.5 Positioning study in relation to accounting and auditing research

The findings from literature on accounting and auditing research, which are concerned primarily with issues relating to financial reporting, are not of direct relevance to the main question posed in this study. Two essential differences distinguish this study and the findings from the reviews of Hogarth (1991) and Libby, Bloomfield and Nelson (2002) of the research literature relating to cognitive research in accounting and experimental research in financial accounting respectively, namely, the nature of the questions posed and the methodology used. Hogarth in his review, *A Perspective on Cognitive Research in Accounting*, reports that auditing research in this area is mainly concerned with auditors' judgements and assessments of risk, i.e., with the quality of outcomes. Libby, Bloomfield and Nelson report upon and classify research which assesses how users of accounting and other financial information report and use information to make judgements and decisions. The methodology used in most of the reported research is experimental with the experimental methods being derived from the disciplines of experimental psychology and economics. In both reviews, the reviewers assess the relevance of the research questions and the results obtained under experimental conditions with respect to conditions likely to arise in the field, i.e., in practice. Libby, Bloomfield and Nelson discuss the greater practical relevance of the experiments conducted in the 1990s as compared with those of the 1960s and 1970s. Hogarth expresses some reservations about the relevance of the research on which he reports:

> "By focussing in depth on particular aspects of audit judgement, CRA [cognitive research in auditing] has made considerable progress over the past two decades. …….. Nevertheless, some progress might have been gained at the expense of achieving a more systematic knowledge of auditing. To correct this imbalance, it is important to develop a more complete description of the different types of judgmental tasks that auditors are required to perform." (Hogarth, 1991, p.258)

On the other hand Libby, Bloomfield and Nelson specifically comment that "[r]ecent studies take advantage of the experimentalist's comparative advantage at disentangling variables that are confounded in natural settings ……" (Libby, Bloomfield and Nelson, 2002, p.776). My research questions are concerned with the minutae of the processes actually used in practice, rather than the quality of task implementation or judgements made in particular circumstances. Hence qualitative and ethnographic methods rather than experimental methods are more appropriate for this study as they enable observation *in situ* and do not prejudge the what and how of participants' use of mathematics in work. However, my assessment of the quality of the performances observed involves not only reporting upon and assessing competence of mathematics in use but also assessing participant competence with respect to achievement of the task in hand. Thus this thesis to the extent it reports and analyses task implementation by auditors provides insights into the process of judgement formation and learning in the field by audit teams, and both relative novices and experts.

In *Learning During Audit Tasks in the Field: A Model, Methodology, and Experiment*, Russo (1997) reports upon first year accounting students' learning as they perform and re-perform a task that involves analysing financial information. Again the methodology is experimental. It involves modelling learning as increasing automation through repetition and a change in content knowledge as a result of doing something unfamiliar. The results of the experiment show improvement in automation (measured as statistically significant) but not a change in participant knowledge content on the second repetition. The experiment did not seek to measure improvement in skill performance. My research analyses the learning observed 'on the job' from the perspective of improving both knowledge and skills. Although it explicitly examines learning with respect to the role of *workplace mathematics*, the description and analysis of some of the critical case studies also illustrate the processes through which both accounting and auditing knowledge and skills are learnt or developed.

3.6 Concluding remarks

The research discussed above has, apart from the accounting and auditing research and some done in the situated cognition and similar traditions, been driven mainly from the perspective of the policymakers in government and business and the Education and Training communities. Policymakers tend to be interested in lists of mathematical knowledge and skills that are used across a wide range of businesses and in the public sector, rather than in the minutae of how mathematics is actually used in and at work, a major concern in most studies based upon ethnographic methods. As a consequence, reforms fail "to address actual workplace needs and to recognise the knowledges workers actually possess, while imposing a regime of pseudo-contextualised skills" (FitzSimons, 2000 and FitzSimons et al., 2003). In particular, I consider that this is particularly so with respect to mathematics education for the 11-16 age group. The exception is perhaps the recent CBI report which is more nuanced with its emphasis on explaining the functional/basic skills employers believe that non-graduates need to have mastered on entry to the labour market.

My main perspective is different. This study is a qualitative study of mathematics in use in a particular workplace; it focuses on how participants work and use mathematics and mathematical practices as they work. A purpose of the study is to provide detailed evidence which can be used with other evidence to assist the development of a realistic functional mathematics curriculum. Consequently in my conclusion, I categorize the mathematics used, the ways in which it was used and the skills that enabled competence, partially to assist policymakers.

I now describe the methodology used for this study.

4 Methodology

4.1 Introduction; influences from anthropology and discourse analysis

My study bridges the academic disciplines of mathematics education, sociology and cognitive psychology. Its methodology is derived from the ethnographic tradition of anthropology, social theories of practice, and theories of situated cognition derived mostly from cognitive psychology as developed within cognitive anthropology.

Methods of data collection and analytical techniques used

An ethnographic approach enables observation of the realization of activities *in situ* (Baszanger and Dodier, 2004) and it:

- provides data about the activities which are the subject of study for analysis;
- avoids *a priori* assumptions about expected results and is open to the discovery of the unexpected; and
- grounds the activities observed in the field – in context.

For these reasons, I chose to use ethnographic methods to collect data, my main method being observation of participants (Cohen, Manion and Morrison, 2000). The style of observation I adopted was non-interventionist (Adler and Adler, 1994, p.378). I sat with the participants as they worked and noted what they did, and collected the texts they used and produced. During phase II fieldwork, observation of participants was supplemented by immersion in the field. Immersion was possible because, even though I spent relatively little time in the field, I have extensive knowledge and understanding of auditing and the culture and work practices of the business observed, having worked within it for over 20 years. Using Gold's (1958) classification my role was that of 'observer-as-participant', as I was known to the team but my interactions with them were not particularly extensive from their work perspective (Le Compte and Preissle, 1993). My study is a study based partially upon ethnographic methods but as a result of my connections with the firm bears traces of classical ethnography.

My initial analysis used Strauss' (1987) theory of action and its methodological techniques of grounded theory. This was only partially successful; it enabled analysis of the activity observed but did not provide access to the mathematical practices embedded within the data. My participants produced/used texts and engaged in discourse, with the mathematical practices being embedded in tasks/subtasks. I, therefore, used textual and discourse analyses to analyse work in action and to uncover the work practices and the resources used therein (Prior, 2004; and Potter, 2004). The analyses focus on the production, content and use of the texts (including, in particular, my observation notes of what my participants did) and on action produced in and through discourse. The unit of analysis is episodic. The level of analysis necessary to get to the mathematical practices is slightly more coarse grained than that used in most discourse analysis.

I chose not to use interviews or extant texts as the primary source of data with one exception[1], as neither gives direct access to action as it is realised in work.

[1] See section 10.7

In the rest of this chapter, I discuss my data collection and analytical processes. Section 2 deals with my relationship to the participants and their working world, section 3 with my fieldwork, and section 4 with the analytical techniques used. In section 5, I discuss ethical issues and the nature of the potential threat to validity from my position as an insider and finally in section 6, I discuss the reliability and validity of data collection and analysis. Some repetition occurs as I tackle each issue separately.

4.2 Fieldwork and my relationship with the participants' working world

4.2.1 Choosing the field

I chose to undertake both phases of the fieldwork for my study in the international accounting firm in which I had been a partner. The pilot study (phase I) was undertaken in the Information, Communication and Entertainment business unit (ICE) while in the follow up study (phase II) I observed part of the audit of a large international bank.

I chose to carry out phase I in a business unit in which I had never worked. Furthermore none of the clients on which I worked in my last five years with the firm were attached to this unit. I discussed the research project with David Ward, the managing partner of the unit. I have known for David for more than 20 years. He readily agreed to me undertaking the pilot project in the firm and he assigned me to an assurance department which deals with clients in media, leisure and tourism, and technology sectors.

Following completion of phase I, I decided to gather further data to elaborate on the findings made in phase I and to explore the creation and use of mathematical models, teamwork and the apprenticeship system in action. I decided to observe these activities by following part of an audit and watching the creation and use of accounting models that incorporated some mathematics. Initially I thought I would need to undertake two further studies; only one was needed. By the time that I came to phase II, David Ward had moved to the Financial Services business unit, so following part of a bank audit in that unit seemed sensible. I selected the audit because I believed that I could obtain good access to that bank; I am acquainted with some of the bank's directors as well as several of the partners who work on the audit. Both the firm and the bank gave the necessary permissions. I discussed my research project with the two partners who ran the bank's group audit. They suggested I followed part of the audit of the loan loss provisions of the UK Bank (the Bank). I agreed to this as I considered that it was likely to generate data of the type I wanted to explore. My last four years within the firm were spent working in the Financial Services business unit and I had advised the Bank but not in the previous ten years. Furthermore I had not previously worked with any members of the UK Bank audit team. After I completed the fieldwork on the Bank audit, I had sufficient raw data on modelling so there was no need to do additional fieldwork.

4.2.2 Reflections on choice of field and methodology

I carried out fieldwork in the firm where I had worked for two reasons: familiarity, and ease and quality of access to participants. In phase II, I chose to follow part of the audit of the Bank for the same reasons.

For the pilot, I chose to use observation supplemented by interview and informal discussion for the following reasons:

- Through observation, I would be able to watch events unfold in real time; interviews as a prime source of data about past actions suffer from being descriptions of past events and inevitably include an element of *post hoc* rationalisation.
- Observation of participants is a way of capturing data on mathematics in use in work within the workplace.
- Initially observation was clearly preferable to interview (or questionnaire) as I was not sure about the nature of the mathematical practices that I would find in the workplace.

As observation proved to be productive in phase I, it remained my prime method of data collection for phase II, but was modified as described in subsection 4.3.3. In particular, I followed the task rather than the participant and, as described above, collected more documentary data.

4.3 Fieldwork

4.3.1 Phase I – the pilot study

The fieldwork for my pilot study comprised extended observations (between 3 ½ and 5 ½ hours each) of five employees as they carried out their day to day work.

I discussed the project in general terms with Tanya Brinkman, the manager of the department to which I was assigned. We agreed that I would observe a number of employees as they worked and that Tanya should select the participants and obtain their permission. We agreed that the profile of the participants would be based primarily on grade within the firm's staffing structures. The five participants comprised:

- an experienced 'in charge',
- an administrative assistant,
- a newly qualified accountant,
- a trainee accountant, and
- an audit manager.

An 'in charge' is an accountant who is responsible for the day to day management of an audit, including the detailed fieldwork. Three of the observations took place in the office and two in the audit room at a client's office. During the course of the observations I sat in on two formal meetings.

Before each observation I introduced myself to each participant and briefly described the project and its purpose. For four of the observations, I arrived in the office at the start of the working day and settled down by the participant's desk before the participant arrived. In the other case, the team had to rearrange themselves so I could be fitted in, as they were working when I arrived. I sat beside the participants as they worked and noted what they did/said. Generally I did not follow the participant if they went to get information from the client, photocopy papers etc. In certain selected situations I accompanied the participants when they went to have discussions with others. After four observations and discussions with Margaret Brown, my supervisor, I decided that I had gathered sufficient data to embark upon preliminary analysis and write a preliminary paper. There are marked similarities between the observations of the different participants and the observations seemed to resonate with the findings described by Lave (1988) in the second part of her book, *Cognition in Practice.* I carried out one further observation; it had already been set up.

I made concise notes of what the participants did and a summary of what they said in an A4 notebook – for extended typed extracts of notes see exhibits in section 5.2. I obtained copies of a few documents on which the participants worked; I only asked for those that I thought would be critical for understanding what the participants did and my analysis. I did not record or video any participant as this would have been very obtrusive and would have altered their working environment radically. When I was a partner I had been part of partner selection panels that sat in, as observers, on role plays and discussions between candidates for partnership. The way in which I made observation notes was similar to that used for the panels – noting down as much as I could about what was happening without thinking too much about what I was seeing.

I typed up the observation notes soon after completing an observation making as few alterations as possible. I also wrote up notes covering the participant's background, the office environment, a summary of the work done by the participant during the observation and background information on the clients' affairs (or equivalent) on which each participant worked. In the case of the first three observations, I interviewed the participant after the observation. Before I did so I wrote a note reflecting on the observation so that I would have something that would capture the pre-interview stage as I wish to based my analysis mostly on what I had observed. In the case of the other two I only talked informally with the participants after the observation. I did not make separate notes of these conversations. I used the information from the discussions to clarify matters in my observation notes.

4.3.2 Reflections on the fieldwork methodology in phase I

Observation of participants proved to be a very productive method of data collection. Consequently much of the data and findings from phase I are incorporated in this thesis and I decided to use the same technique in phase II.

Observing part of a day's work enabled me to make an assessment of the mathematics that is embedded in an ordinary day's work. From the viewpoint of observing the use of *workplace mathematics* in context, the 'day in the life of' approach had three significant disadvantages: I often only observed a participant undertaking part of a task; insufficient reference was made to the participants' source texts; and my observations provided insufficient insight into the role of teamwork and learning. In phase II, I sought to overcome these disadvantages by using the task, not the person, as the sampling unit. I also collected copies of the working papers that participants produced and more of the source documents they used as they worked.

In my pilot study, the competence of my participants was noteworthy. I consider that this was probably partly due to the way the participants were selected. Tanya sought willing recruits; it is reasonable to assume that the selection process tended to produce volunteers who were both confident and had a relatively high degree of competence. This problem of self selection did not occur in phase II because the international bank's audit partners chose what I should observe. Tom, the audit partner in charge of the UK Bank audit, was enthusiastic about the project. I gained the impression that the other members of the audit team were less so. Consequently in phase II I observed more audit work where the participant was not sure about either what to do or what was wanted.

4.3.3 Phase II – the Bank audit project

The fieldwork comprised observing the audit team perform five loan loss reviews and part of the review of the vacant space provisions for City offices. In the case of four loan loss reviews I observed most of the work from start to finish. In the case of the other, I started observing at the review stage.

I met Tom and the audit team at the Bank's premises and discussed in outline my project and what I wished to do. Tom briefed me about the loan loss review, which was then in progress; Eric, the audit in charge, had only a few more credit reviews to carry out. It was proposed that I should observe him carry out four of the outstanding reviews and follow those through the review process. It was agreed that I would attend two client meetings (these had already been set up to enable Tom to obtain an overview of the Bank's loan portfolio and the bad debt provisions) and the review meetings. Tom consented to the taping of the review meetings but felt it would be inappropriate to tape the client meetings. I also observed part of the process of creating the Audit Committee Paper as the loan loss review was discussed at length in it. I added a fifth loan loss review to my sample after the first review meeting. I had suggested to Eric and Cliff, the audit senior manager, that in that meeting they should talk about two loans that I was not concerned with to get them used to working while being observed and taped; the first discussion – on Kookaburra – was of such relevance to my research that I decided to add it to my list of observations. It was also agreed that I would observe Ramesh, an audit assistant, as he carried out part of his work on accruals and provisions. Ramesh reported to Jerome, one of the three managers on the audit team. All the observations took place on the Bank's premises and I was assigned a desk with the audit team for the rest of the year end audit. I was free to come and go as I pleased.

My observation techniques were similar to those used in phase I. However I systematically collected a wider range of data. When a participant worked alone I sat beside them and made detailed observation notes (e.g., exhibits in subsections 8.2.5 b) and c)), obtained electronic and hard copies of their working papers as they were drafted (e.g., I have six electronic copies of the incomplete Wallaby working paper at different stages) and collected copies of more of the source documents used as the participants drafted their working papers. When I arrived on the second day to observe Eric, he had attached a second monitor to his PC, so I would not have to look over his shoulder as he was working; this innovation eased and improved observation considerably. I made brief notes of and taped all review meetings. I made observation notes of other meetings and obtained copies of key papers that were referred to at those meetings. I also taped relevant parts of the Audit Committee Paper drafting meeting. As I sat with the team for about 10 weeks for about three days a week I got to know them quite well; this meant that I did not interview them but simply asked for clarification at convenient moments. They were very helpful.

I typed up my observation notes and made transcripts of the tapes soon after completing an observation. I did this as I sat with the team. I made as few alterations as were necessary for future understanding. I have since improved the transcripts of those parts of the tapes that have been subject to detailed textual analysis (that is with the exception of one which was inadvertently wiped after I made the first transcript). I also have notes covering the participant's background, the office environment, the task in hand and relevant background information on the Bank and their customers.

4.3.4 Reflections on observations in the field

Although I did not specifically seek out mathematical activity, I expected to observe it, particularly as the observations were to be carried out in an audit department and on an audit.

As I watched the participants work I made choices. I described the participants' working environments briefly and noted what they did and, to a lesser extent, what they said. As they worked they tended to explain what they were about to do, or occasionally what they were thinking or doing. I noted these explanations briefly. When I typed up my notes, I made minimal amendments. I asked participants to read some of my notes/commentaries, particularly where I was uncertain about something, but I did not ask them to comment extensively on what they had done and why. I tried to record what was happening without making too many judgements. These would follow later when I analysed the notes of the observation. In this way I tried to preserve as near a contemporaneous record of what I observed (i.e., selected) as was possible; inevitably my efforts were undoubtedly biased towards capturing what I perceived as being mathematical.

It is to these records that I turn, and return, as I analyse. Consequently my analysis is heavily dependent upon the quality of my record of what the participants did and the choices that I made as I recorded the observations.

4.4 Analysing data

4.4.1 Analytical techniques used

I used a number of different techniques for analysis. The process is described as it evolved. In practice, the process was iterative. Most of the techniques used are demonstrated as I discuss my findings.

1) I started each phase by describing one observation – in the case of phase I a day in the life of Joan, and in phase II the Wallaby loan loss review. The descriptions were derived from the data collected and emphasis was placed upon the participants' actions, the nature and purpose of the auditing tasks undertaken and the obvious mathematical practices. Tom and Eric read my Wallaby paper and

confirmed that it was a fair record. The purpose of this exercise was to understand what had happened and to start developing descriptions of the practices observed.

2) In phase I, I first analysed my observations using the techniques of grounded theory. The analysis was restricted to what I observed during the formal observation of participants. My methodology follows closely that described in Strauss (1987). On my third attempt I developed a system of coding which seemed to be reasonably flexible and robust. I used the grounded analysis results to write up an account of Joan's working day. Towards the end of phase I, five core categories emerged: task, resources used, own knowledge and skills (including a subcategory of thinking skills, such as using mental schema/models), *workplace mathematics* (or embedded mathematical practices) and facilitation. I checked that data collected in phase II could be analysed in a similar manner. Grounded analysis threw light on how the participants' working days were structured, the importance of task/subtask as a unit for analysis and participants' own knowledge and skills but little was generated beyond focusing on the person engaged in activity within the immediate setting. I needed different analytical frameworks to enable analysis of the relationship between task/subtask and the embedded *workplace mathematics,* and the contribution to my participants' endeavours of teamwork and of tools/artefacts which incorporate the results of past cognitive effort.

3) During phase I, I reviewed my notes of some of the practices observed, described the tasks in which the mathematical activities were subsumed and the mathematical activities themselves. Textual analysis thus emerged as my key method for revealing *workplace mathematics* embedded in work practices without losing the significance of context.

4) I looked for and found *workplace mathematics* in the same way that Nunes et al. (1993) searched for and found *street mathematics.* I described the *workplace mathematics* found and formulated a preliminary working definition. I, then, considered and answered the question, 'How do I recognise participants using mathematics?' and as a consequence elaborated my definition of *workplace mathematics.* To answer this question, I was forced to address the question, 'What is mathematics?' – see discussion in section 2.3.

5) **Methodology for textual analysis used in phase II** Discussions in qualitative analysis workshops led by Mike Askew suggested that the unit for analysis was either a task/subtask or a short conversational episode. Grounded analysis confirmed this to be the case. I analysed tasks/conversational episodes through observation notes, transcripts of meetings and the texts produced and used by my participants. I reviewed all the episodes chosen several times, always seeking to:
   a) understand and describe the task in hand from a commercial perspective;
   b) identify *workplace mathematics*; and
   c) consider how the *workplace mathematics* was used;
   sometimes considering:
   d) how the task and *workplace mathematics* fitted into larger contexts and were influenced by practices, available resources and settings;
   e) the interplay between the individual and the team (and occasionally others); and

f) the knowledge and skills brought to the task by the participants;
and occasionally speculating:
g) about a participant's thought processes.
Tasks a) to c) were usually carried out in that order, though on many occasions I would go through the cycle several times, as repetition often brought more to the fore. The textual analysis was preceded by breaking the tasks/subtasks into units for analysis. Episodes selected for analysis were chosen to illustrate different aspects of *workplace mathematics* observed. The analysis was informed by particular theoretical frameworks, namely Luhmann's analytical approach to observation, theories of practice, Hutchins' theory of distributed cognition, Lakoff's concept of idealised cognitive models and towards the end of the phase, Wittgenstein's concept of knowledge acquisition through rule governed practice. Thus, I analysed the work practices observed to describe and categorize the *workplace mathematics* observed and to show how participants do *workplace mathematics.*

6) I then compared my findings relating to *workplace mathematics* with findings from key texts which have influenced this study – Lave (1988), Nunes et al. (1993) and Hutchins (1995).

7) Finally I conclude by categorizing and summarizing my findings and briefly considering their implications for functional mathematics and further research.

4.4.2 Reflections on analysis of data

My analysis of the data collected is supplemented by my tacit knowledge and my memories of observing the participants themselves and of working in the same room with them, formal and informal discussions, and other data.

In both phase I and II, I chose not to use certain observations. From the phase I observations, I discarded all the material in the observation of Holly, the audit manager, that involved discussion of the accounting treatment of the option within a purchase agreement because Holly (and the partner to whom she was reporting) kept drawing me into their discussions and so compromised the data collection. From phase II, I completely discarded one loan loss review because Eric's working paper was a rough draft, which needed a lot more work when the audit team ceased working on the review because the Bank had decided to sell the loans before the year end.

I sought to ground my analysis in the practices observed by starting most of my lines of enquiry, theoretical and practical, with evidence direct from my observations, the key exception being Eric's re-analysis of the Wallaby half year results (section 10.7). I have noticed that, as my analysis goes through several iterations, my explanations, if I am not careful, can easily be transformed into something I expect or would like them to be. I guard against this by continually returning to verify my arguments as they develop against the primary data. However through the process of analysing the texts I reconstruct what my participants did, rather than just describing what I saw them do.

My analysis is biased. I expected to find work practices which resonated with the theory of situated cognition and participants' using mathematics; I did. Relatively little emerged about the wider contexts of the audit and the audit team, the office, the firm's and clients' affairs and the business world. This is noteworthy and probably inevitable as I was looking at the data with bias – the bias of my research questions. My method of textual analysis was focused to give insight to those questions. Equally my data could have been used to analyse computer use or work and management structures.

My data collection and analytical techniques did not capture all the *workplace mathematics* that my participants used as they worked. For example, when they read texts with substantial mathematical content for understanding and that understanding was not manifested by action, it was not possible to assess what was done. I observed Eric reading a substantial accountant's report on the future of the steel industry produced in relation to one of the loan loss reviews. He marked up the report extensively with a highlighter. However from the evidence collected it was impossible to determine if he used *workplace mathematics* as he read and if so what, even though he almost certainly did. Furthermore I have not analysed source texts or the tools and artefacts used in order to tease out the *workplace mathematics* that contributed to their creation; this is a subject for another study.

4.4.3 The role of interpretation; second order observations

I specifically sought to keep my observations 'out of the head'. This has had important consequences for the narrative captured and my analysis. Where my participants were silent as to their thought processes, which was most of the time, and they are not evident from manifested action, the interpretation of those thoughts is my interpretation. Sometimes I used clues provided by my data – particularly contiguous actions. Sometimes I made inferences as I interrogated the data closely. Sometimes I relied upon my tacit knowledge of the working environment. It also turns out that this heuristic approach is critical to my analysis of the mathematics in use. Generally the participants explained or talked (to me or to colleagues) about the task in hand or talked through some aspect of it. They rarely talked through what they were doing on a step by step basis. The mathematics used tended to be embedded in tasks with the result that my analysis of the activities observed is dependent upon my substantial tacit knowledge, my interpretation of the primary data and assumptions about participants' unarticulated thoughts, as well as the data itself.

4.5 Ethical issues

4.5.1 Compliance and confidentiality

David Ward and I agreed that I would observe the firm's rules with respect to confidentiality in relation to clients' and the firm's affairs, and also that the normal research rules regarding participant confidentiality would be observed. I agreed with the international bank's chief executive that I would observe the same code of conduct with respect to the Bank's affairs. I also acknowledged that I would need to take care to ensure that I did not breach the laws relating to insider dealing.

I agreed with David that, if, in the course of my observations and other research, I came across anything that was concerning from a business perspective then I would raise the matter with an appropriate person within the firm. David left it to my discretion and experience to decide upon the appropriate person.

4.5.2 The insider as researcher - reflections

After discussions with Margaret Brown, I carried out my fieldwork in the firm where I had been a partner enabling me to bring my knowledge and experience of business, in particular and in general, to bear on the research. We assumed that the access that I would gain would enable effective observation of participants. Being a privileged insider inevitably has major influences upon research. It enables me to create a 'thick description' (Geertz, 1973) of the participants and their workplace but it is not without drawbacks.

Influence of the role of insider on the role of researcher I am a privileged insider – by virtue of my previous position and the number of people I still know. I am an insider in that I am able to share understanding of overall and individual business objectives, in many cases down to the level of the individual tasks into which assignments are broken. I am an insider with respect to the cultures of the firm and its employees, and to the extent that I want the firm to succeed, to be respected in its marketplace and in the community, and to be a good place to work. I know it won't always be. I also have over 20 years experience of working in the firm.

Influence of the role of insider on the researcher in the field To compensate for the inevitable bias in collecting data, I decided to rely mainly upon observations, and only to use information gained in interview and discussions to provide background or necessary elaboration. I strove only to record what the participants did/said. Inevitably as I observed, I thought about what was going on and my prior experience affected what I noted. However the act of recording was often sufficiently engrossing to prevent me from thinking/theorizing.

Influence of the role of insider on the observations As I observed I acted not only as a researcher but also as an experienced accountant who had worked within the firm. It is these latter experiences that help me to give meaning to what the participants did. This inevitably gives rise to bias but it has had a beneficial effect on the richness of my findings. With respect to the participants there is an important issue about which I could do relatively little; the participants were aware of and affected by my previous standing within the firm's hierarchy. Reassurance and observing for a relatively long period of time seemed to put them at ease, and three told me that they forgot that I was there when they were concentrating hard.

Influence of the role of insider on analysis and synthesis I act primarily as an insider when I act as biographer of the world in which the participants worked. In particular, I use my tacit knowledge of the firm and accounting extensively in writing about all background contexts. When I analyse and theorize about the observations, I act primarily as a researcher from within the education research community but inevitably my experiences as an accountant have substantial influence on my findings and theorizing.

The influences derived from being an insider could be a major threat to validity. I have tried to limit the threat by being explicit about the nature of the influences, taking some actions to achieve some objectivity in data collection and analysis, and by reflecting rigorously upon the impact of insider status. Nevertheless, the advantages of being an insider provide the opportunity to analyse in depth the mathematical practices in a workplace (outside the education community), so that the pitfalls arising from insufficient knowledge and access are avoided, and the tendency to decontextualization of the mathematical practices are reduced.

4.6 Reliability and validity of data collection and analysis

4.6.1 Reliability

Issues of reliability and validity present problems for qualitative research based upon observation of participants. Reliability, which is a prerequisite for validity, is concerned with consistency and replicability over time, over research instruments and over groups of participants. There are sufficient extended observations within this study to enable some claims with respect to reliability. There were 10 principal participants and 11 extended observations, most of which involved more than one person. Observations were made over two phases in two business units, albeit within the same firm. Several types of data sets were used for analysis in phase II: observation notes of participants' working, transcripts of recordings of review meetings, working papers and notes produced by participants and, only for corroboration, source documents used by participants as they worked. In phase I, observation notes of particular participants working was the main source of data. Observations of individuals working alone tended to provide evidence of work which involved the writing of a text or work which was documented in written texts, while transcripts of review meetings tended to provide evidence of work in and through discussion. The analyses discussed in chapters 5 to 11 show some consistency over the observations and across individuals. However as the data was collected in one business and collected and analysed by one observer, it is not possible to make strong claims for reliability based upon this study alone. Claims to reliability are, however, strengthened by reference to the similarities between findings in this study and those in other studies (see below) and the presentation of sufficient evidence for the findings to allow them to be measured against other studies of workplace mathematics.

4.6.2 Validity

Invalid research is worthless. It is, therefore, necessary to set out claims for validity for this study. Cohen, Manion and Morrison suggest that

> "in qualitative data validity might be addressed through the honesty, depth, richness and scope of data achieved, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher." (Cohen, Manion and Morrison, 2000, p.105)

They also state that validity should be seen as a matter of degree. In this chapter, I recognise threats to validity and show how I tried to reduce those threats with respect to my role as an insider, participant selection, and data collection. I also recognise that I observe/analyse the data with the bias of my research questions and expectations and in the process reconstruct what was observed. My main method of reducing the threats to validity of analysis has been to use and continually return to raw primary data, namely my records of the observations themselves. All the substantive examples discussed in this thesis, except for Eric's re-analysis of Wallaby's half year results, are based upon records of observations. My analysis of Eric's re-analysis of the Wallaby results is based upon the two texts (appendices 4 and 5) he produced after a private discussion with Tom (not observed). This incident of near the job / on the job teaching and learning is such a critical event (Flanagan, 1949; and Wragg, 1994) from the perspective of my research questions that I report and analyse it in depth[1].

Given the importance of validity, it is worth looking at it from another perspective. According to Cohen, Manion and Morrison, Maxwell (1992) argues for five kinds of validity in qualitative methods:

- descriptive;
- interpretive;
- theoretical;
- evaluative; and
- generalizability.

The discussion above on reliability and validity broadly covers what Maxwell means by descriptive validity.

Interpretative validity is the ability to catch the participants' intentions, meaning and interpretations. This has been partly achieved by establishing the centrality of task/subtask performance through grounded analysis and reference to contemporaneous records and observations notes (including transcripts of tapes) for participants' intentions, understandings and interpretations. In several cases I asked participants to confirm/clarify my understanding of the task in hand or of the accounting or auditing principles in use. My prime purpose is to use the observations to uncover *workplace mathematics*. My participants recognised some of their activity as mathematical activity but much was not so identified because it was embedded in another perceived activity. My use of the data, thus, arguably runs contrary to the principle of interpretive integrity. I have tried to reduce one of the main impacts of analysing against the grain of the task in hand, namely decontextualisation, through the use of an analytical method that reveals the relationship between the task in hand, the work done and the embedded mathematics.

Theoretical validity is concerned with theoretical constructs on which explanations are based. The underpinning constructs relevant to my research are discussed in chapter 2. In particular, Hutchins' theory of distributed cognition provided a framework for analysing work in an institutional setting. My search for *workplace mathematics* and explanations as to what it is and how it is used were directed primarily by my main research question with my main analytical methods being that of textual and discourse analyses.

By evaluative validity Maxwell is referring to the application of a framework judgemental of the research. I built no such framework into the design of the study. Arguably this was not necessary. My main research objective is to describe and show how my participants do *workplace mathematics* as they work. Observation of participants provided direct access to many but not all work practices – see subsection 4.4.2. My key analytical technique during phase II was iterative textual and discourse analysis. Thus I have allowed my observations, the participants' texts and contemporaneous discussions to reveal the what, why and how things were done.

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[1] See section 10.7.

Key to the validity of this study is the possibility of its generalizability, i.e., the view that the findings generated may be useful to those researching workplace mathematics generally and to those involved with the role functional mathematics should play in secondary school mathematics within the UK and elsewhere. In writing this thesis I have adopted a descriptive method that reveals in detail how my findings derive from primary data so that the results can be compared against the results of other studies. In chapter 12, I compare my findings with some of the mathematical practices observed and discussed by Lave in *Cognition in Practice* (1988); Nunes et al. in *Street mathematics and school mathematics* (1993); and Hutchins in *Cognition in the Wild* (1995).

I now turn to my analysis.

5 Searching for *workplace mathematics*

5.1 Searching for *workplace mathematics* – expectations and methodology

In this chapter, I examine data from phase I and look for and describe practices that may be exemplars of *workplace mathematics.* In the next chapter I discuss how I recognise participants using mathematics and then I develop a definition for *workplace mathematics.* In chapters 7 to 12, I use data mostly from phase II fieldwork to illustrate how *workplace mathematics* is performed and used in practice.

As can be seen from my original research proposal I had expectations of what I would find:

> "To make the research manageable, I intend to constrain the area of enquiry in certain respects:
>
> - By mathematics, I shall primarily look at the mathematical skills employees use. I do not intend to restrict the meaning to commercial arithmetic as I expect to find employees using statistics, mathematical modelling, algebraic formulae, sets and logic, and informal geometry, even though they might not describe them as such. I would also like to determine whether or not the processes of mathematical investigations, conjecture and proof inform their thinking.
> - Non-specialist employments[1] do not include employments for which a degree in mathematics or a physical science is normally a prerequisite." (Source: original research proposal.)

In subsequent chapters I do not refer to commercial arithmetic, as it implicitly encompasses arithmetic in use in commercial practices. In this thesis, I distinguish between arithmetic and its usage.

My participants met the employment constraint and used most of the mathematics and mathematical techniques listed in the above quotation. However much of the mathematics in use was not immediately recognisable as such as it was embedded in/interwoven into work practices. A preliminary review of some of my phase I field notes indicated that in the course of their everyday work participants used a variety of mathematical practices – being mostly that of arithmetic and the use and construction of financial mathematical models. I analysed the data more closely to uncover a clearer picture of the mathematical practices used. To do this, I selected suitable extracts from my observation notes, highlighted mathematical activity, described the substantive activity and the nature of the mathematical practices embedded[2] therein and used this analysis to generate a working definition of workplace mathematics.

5.2 Searching for embedded mathematical practices

5.2.1 Introduction

I have selected one or two episodes from four of the five observations from phase I to illustrate activities that appear to incorporate activity of a mathematical nature. The

[1] This serves as a definition of 'non-specialist employment.'
[2] 'Embed' means fix firmly in surrounding mass. (The Concise Oxford Dictionary, 1982)

other observation – that of Joan – was used for other purposes. Episodes were chosen partly for their narrative quality and partly for the nature of the mathematical practices that appeared to be embedded in the activities observed.

Extracts of my field notes are boxed in tables and are typed in 10 point. Bold type usually indicates where numbers/mathematics appear on an initial judgement to play a part in the work done. Italics are used for summarized notes or for information inserted to aid clarity.

5.2.2 Holly, an audit manager, reviews the accounts of a subsidiary

During the course of the day on which I observed Holly, her main task was to progress the issue of how the put option in a joint venture agreement relating to C group should be presented in the 20F, a SEC (Securities Exchange Commission) return, which had to be filed that day. She also reviewed the UK statutory accounts and audit files of two subsidiaries of C and dealt with a number of queries. Work on the put option took priority. The extract covers the review of one set of accounts.

Exhibit 1 – Extract from my observation notes; Holly reviews C1's accounts

9.55am	Holly takes C1 draft accounts from the working papers file *The 2001 accounts comprise the directors' and auditor's reports, the profit and loss account (P & L a/c), the balance sheet and the accompanying notes.* Takes out calculator Turns to the debtors section in the working papers file **Starts reviewing work done by senior** **Looks at lead schedule which has breakdown of the accounts figure for debtors** *...social conversation with colleagues...* Reads work done schedules on working papers file **Quickly looks through schedules supporting lead schedule in debtors section** Goes to draft accounts
10.05am	*Reviews directors' report – this includes proof reading, redrafting 3 points,* ***checking numbers to and extracting them from other schedules in the draft accounts*** *and writing two review notes on the draft accounts.* *There were three minor interruptions during this review.*
10.20am	Reviews auditor's statement Redrafts heading Redrafts another sentence in report **Turns to the P & L a/c** Corrects a number of minor typing errors **Checks numbers to backing schedules** **Checks numbers using calculator** **Looks at balance sheet** **Checks numbers to backing schedules** **Checks numbers using calculator** Has a review point on balance sheet Takes out set of C Group's published accounts and looks at presentation of point Redrafts point on C1's balance sheet Looks at notes to the accounts Redrafts point Writes review note re 'going concern' on accounts

10.30am	Phone rings. *Gathers together papers and goes to meeting with partner re the put option.*
12.35pm	Returns to review of C1's accounts Continues review of notes, making minor changes Makes note of review points on accounts **Checks figures to intercompany balances section of working papers file** **Seeks to understand the amounts due to C1 from parent and subsidiaries** *...interruption – about 5 minutes...* Returns to reviewing C1's accounts **Tries once again to see if satisfied that intercompany debtors are recoverable by looking in working papers file** Takes out manager's pad and heads 'HRB review points' Holly tells me that **she is writing a detailed explanation of her main review query to help the in charge. She is concerned about the legality of the dividend for the year as it depends upon recoverability of debt from group company. She, Holly, is not satisfied that the in charge has satisfied herself on this point** Continues to review notes to the accounts Puts question mark beside a note Looks up Companies House records of C1 on her laptop – *using the internet.* Puts note on the accounts **Checks numbers in the P&L a/c note** **Uses calculator to add total** Refers to note in C6's accounts **Uses calculator to total numbers** Notes review point on accounts
1.09pm	Finishes review Files accounts and review points schedule

The task Holly's task was to review a draft set of annual accounts which had been prepared by the client and had been audited and reviewed by the audit in charge. The process noted above comprised reading the accounts to check that they fairly reflected the company's financial position, in addition to proof reading, checking numbers to backing schedules and/or underlying records (the audit working papers in this case) and checking totals and subtotals for accuracy.

Mathematics in the task During her review of the accounts of C1, Holly engaged in several different mathematical practices. Firstly, she read and reviewed C1's balance sheet and profit and loss account (P& L a/c), extracting meaning from them as she progressed. Numbers, in terms of monetary values, are core to the meaning of these accounts, as is written text elaborating and complementing the numbers. Secondly, Holly checked numbers were consistent throughout the accounts. Thirdly, she also checked the arithmetic accuracy of totals and subtotals, always using a calculator. Fourthly, Holly used the mathematical structure of the balance sheet – the total of the values attributed to the assets balancing (i.e., equalling) the values attributed to liabilities and the shareholder's interests – to make a deduction about the dividend shown as payable. She had a major concern. Was the dividend legal? How did she become concerned? Through her reading of the numbers in the draft P& L a/c and the balance sheet, she inferred that the dividend was partially or wholly funded out of assets comprising the amounts due from other group companies. The logical deduction was made using the mathematical structure of the balance sheet and the information contained therein. This led her to reviewing the audit working papers. This task included reading the tables showing the make-up of the intercompany accounts. When she was unable to satisfy herself that the work noted as done provided sufficient assurance that the debts due from group companies were recoverable, she had concerns about whether C1 was a 'going concern' and so wrote a review note for the in charge setting out her concerns.

Finally, Holly made use of the organisation of the financial numbers in the working papers file and the firm's standard alpha-numeric referencing system that ordered them. Numbers from the year end accounts together with those in supporting analyses are located in the file using the referencing system; as are all working papers. A lead schedule in the working papers provides a breakdown of a balance sheet balance. Key numbers in the breakdown are referenced to and back from further backing analyses. Work done is noted on the schedules or supporting schedules and both are referenced to other audit evidence. Usage of the ordering properties of the alphabet and numbers of the referencing system is essential to enable effective navigation through an audit file.

5.2.3 Gary, a second year student, checks the client's schedule of directors' remuneration

Gary worked on two main tasks during the observation; auditing information in the draft financial accounts of T relating to the directors' remuneration note and other profit and loss account notes, and reviewing the fixed assets section of the audit which had been prepared by Richard, a first year student.

A schedule of the directors' remuneration prepared by the client formed the basis of the information in the note to the draft accounts and Gary's work.

Table 1 – Structure of client's schedule of directors' remuneration

Name	Salary/ bonus £	Bonus £	Benefits £	Subtotal £	Employer's pension contributions £	Total £
MD	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX
OD1	XXXX		XXXX	XXXX	XXXX	XXXX
FD	XXXX		XXXX	XXXX		XXXX
OD2	XXXX		XXXX	XXXX	XXXX	XXXX
OD3	XXXX		XXXX	XXXX		XXXX
OD4	XXXX	....	XXXX	XXXX	XXXX	XXXX
Total	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX

Exhibit 2 – Extract from my observation notes; Gary's work on the directors' remuneration

9.40am	.... G goes across to table to get working papers file Looks up schedule headed 'Pay Reconciliation Jan-Dec' **Looks at the client's schedule of directors' remuneration (RN)**	

	Opens his calculator and gets pencils/biros out ***Uses calculator to check totals of all columns and rows finding a difference of one, which is noted on RN.*** *.....minor subtasks....* ***Checks directors' emoluments schedule to Pay Reconciliation Jan-Dec for all directors other than MD. In the case of FD and OD3, creating the audit trail requires deduction of pension contributions from the salary figures in RN to enable reconciliation. Reconciliations noted on RN and referenced to working papers. G uses calculator to determine differences.***
10.15am	Goes to ask client question about salary of MD G returns and tells observer "Director's salary recharged by shareholder. T **picks up 80%"** G has brought back file with invoices in it. He starts trying to reconcile invoiced amounts with the figures in RN **Uses calculator to 'total' amounts on invoices** **G tells observer "Pay increased to 400k. Invoiced at the rate of 300k during the year. Difference invoiced after year end"** **Looks at monthly invoice, uses calculator and talks to CI, the in charge, as he tries to reconcile numbers to RN** ***Calculation from invoice amount*** **19*12=228k** ***Expected rate*** **80%*300=240k** ***Both calculations done on calculator.***
10.30am	Goes to ask client about difference Comes back .... G has found out that the salary number is composed of two figures (not one) on invoices and summary schedule attached to front of invoices **Looks at and tries to understand summary schedule attached to front of invoice** **Uses calculator to try and reconcile figures**
10.40am	Goes out of office, *presumably to see the client,* and comes back Looks at RN Runs finger over numbers Says "it's not getting there...."
	Goes to get client to explain exactly how figures tie up
10.46am	Comes back and says "We're there.......MD was only here from May.....hence only 8 months........**and so figures are right"** **(2/3*(80%*400))** *(that is, total in RN)*
10.50am	Goes to photocopy evidence - summary and copy invoices Comes back with photocopies Sorts *....social conversation.....* Notes work done on MD on RN Thinks G talks to observer about which figures should appear in the financial accounts G continues noting evidence on RN. **References papers and notes reconciliation of amounts** *....minor interruption....* G marks up photocopy of invoice Writes explanation of how salary increased and how increase paid in a lump sum after year end **Writes out detailed calculation of how figures agree to accounts figure** ***pre bonus and benefits as shown in RN*** **£240,000*8/12 + (0.8*100,000)*8/12 = £213,333**

11.00am	Assembles RN plus backing schedules which will serve as evidence
	Thinks and talks to observer about pension contributions and benefits
11.06am	"I'll speak to CI about pensions and checking benefits"
	Realises that he needs to check bonus for MD
	Together G and observer dig out invoice **and do calculation**
	G does on calculator and [observer does in her head]
	80%*80k=64k
	No notes on how G ended task but he started new task at 11.10am

The task Gary's task was to audit the directors' remuneration note. In the above extract, Gary can be seen carrying out part of the task. In particular he checked the arithmetic integrity of the remuneration schedule, the directors' salaries to records in the client working papers file and the pay of the managing director to intercompany invoices. In verifying the managing director's salary, Gary carried out two aspects of the task simultaneously rather than separately; he checked the total of the amount charged to the company, T, by one of its shareholders and calculated the amount that should have been charged simultaneously. After he had initially found out from the client how and what the managing director was paid, he tried to check that the monthly charges made by the shareholder (who employed the managing director) reconciled with the total charged to T *during* the year. When he was unable to reconcile the figures, he had three further discussions with the client to get more information to clarify his understanding, in the process finding that the monthly charge comprised two amounts and that the managing director had only worked for the company for eight months of the year. He then calculated the *total* basic salary (including the increase paid after the year end) that should have been charged and recognised that he was able to reconcile it to the amounts charged to T through intercompany invoices (and the attached summaries). Gary then assembled his audit evidence and wrote up his findings. Almost as an afterthought, he realised that he needed to check the figures relating to the bonus.

Mathematics in the task Gary used many different mathematical practices routinely. He read and understood the numbers in the remuneration schedule. He checked all totals in the remuneration schedule using his calculator. It is noteworthy that Gary performed all calculations on his calculator. He checked the pay of the directors, other than the managing director, to audit working papers. In two cases, he created a written record of how the salary in the remuneration schedule reconciled to that in a copy of the pay records; this involved deducting the personal pension contributions paid by the company on behalf of the directors (extracted from copies of company records) from the figures in the remuneration schedule. In other words he proved the integrity of the figures used in the accounts.

When Gary started checking the managing director's salary, he did not have sufficient information to carry out the checking. We see him gradually gathering and understanding the information necessary to make sense of and verify the salary in the remuneration schedule (lines 15-16, 20-21, 30, 40-42). Here Gary used mathematical practices in a qualitatively different way. He used exploratory[1] calculations to help him make sense of the information (lines 19, 24, 25, 34 and 42). He calculated the pay

[1] Defined and discussed at length in section 10.3.2.

that he thought should have been charged through the year and tried to reconcile it to what he assumed was the monthly charge. When the calculations did not equate, he sought more information from the client until he understood the position when he said. "We're there. The managing director was only here from May. Hence [he was] only [here for] eight months [of the year]........and so [the] figures are right." He then calculated, 2/3*80%*£400,000, so confirming the calculation of the number in the remuneration schedule. When Gary wrote up his notes, he uses the following calculation,

> £240,000*8/12 (in an earlier calculation he had reduced the £300,000 to £240,000) + (0.8*£100,000)*8/12
>
> (i.e. amount charged in the year + amount charged after the year end)

as proof that the recharge was correct. This is an idiosyncratic calculation which reflects both the way the remuneration was charged to T by its shareholder and how much T should have been charged. Gary used mathematical practices to help him to make sense of financial information; they helped him both structure his understanding and resolution of two issues – how much was charged to T and how much should have been. The calculations he created to show that the salary recharge was correct reflected the way he re-composed the charge in order to reconcile it to the charges made by the shareholder. The calculator and the calculations he created scaffolded his understanding of the charge and in turn structured his explanation thereof.

Gary referenced his working papers using the firm's referencing system.

5.2.4 Sasha, a newly qualified accountant, audits the systems for accounting for transmitted programmes and has a progress meeting with T*'s management accountant

On the day of the observation, Sasha and her assistant, Jonathan, were in the middle of the fieldwork for the audit of T*, a television company, for the nine months ending 30th September 2001. In the morning, Sasha worked on three main tasks: testing the systems for transmitted programmes to confirm their reliability with respect to generating year end balance sheet figures relating to programme costs; reviewing the work of Jonathan; and preparing carefully for a meeting with Sonjoy, the management accountant of T*. In the afternoon Sasha had a meeting with Sonjoy, the purpose of which was to discuss and progress Sasha's (and some of Jonathan's) audit queries.

The observation took place at the client's premises in the audit room in the morning and in a meeting room in the afternoon.

The following four exhibits are based upon my observation notes.

Exhibits 3 to 5 should be read in conjunction with table 2. Before lunch, Sasha set up spreadsheet models for exemplars of programme accruals and year end stock.. The spreadsheet for a commissioned programme accrual, table 2, was one of nine. I observed Sasha working on six of the sheets.

T*

20

PERIOD ENDING 31 SEPTEMBER 2001

PROGRAMME ACCRUALS

		Accrual
		£
Commissions	S	355,000
		£
Per CODA		
Invoiced		0
Transmitted		(355,000)
		(355,000)

Per Genersys			
Total Contract Value		355,000	
Transmitted	2 Episodes @ £50k	100,000	Per Genersys episode value varies.
	3 Episodes @ £27k	81,000	
	1 Episode @ £80k	80,000	
	2 Episodes @ £40k	80,000	
	1 Episode @ £14k	14,000 ?"	
		355,000	

Per Contract		
Total fee		435,000
No. of episodes		10
Fee per episode		
	First 2 at £50k	
	Next 3 at £27k	
	Next 4 at £80k	
	Final at £14k	
Permitted runs per episode		3

Per Contracts Dept one episode has been cancelled. This episode was costed at £80k which is now not payable, although a small payment will be required of which the amount has not yet been agreed.

This explains why the Genersys system shows 9 episodes transmitted, whilst the contract was for 10 episodes.

The difference between the contracted total fee of £435k and the Genersys total fee of £355k is the £80k relating to the cancelled episode which has been taken off of the Genersys system.

Exhibit 3 – Extract from my observation notes; Sasha creates spreadsheet for commissioned programme

12.30pm	Sasha gets copy of client's programme accruals schedule
	Opens up computer
	Starts creating new spreadsheet
	Types;
	    Heading
	    Subheading 'Sample'
	    Commissions *(1)*
	    XTV 1 acquired *(2)*
	    XTV 2 acquired *(3)*
	    Sports *(4)*
	    Repeats *(5)*
	Types name of single sample under each sub-subheading
	In column 4 – types 'Accruals'
	Puts in numbers for five selections from programme accruals schedule
	Looks to and fro
	Types in note 'Covered in work on stock' against 2 items
	Sets up separate schedule for each of the three others – headings first then copies sections for each choice
	
	Commissions schedule – see table 2
	Types in column 1 CODA, invoiced and transmitted
	Finds invoiced and transmitted values on CODA records and types in
	Sets up total and calculates difference using autosum function
	Looks up Genersys printout
	Highlights relevant numbers in orange
	Goes to stock spreadsheet to copy layout – Genersys, total contract value and transmitted columnar layout
	Puts in contract and transmitted values from Genersys records
	Puts note beside transmitted cell
	Reads contract and highlights points in orange
	Puts in episode price and number from contract onto spreadsheet - in process of doing this refers to schedule of costs attached to the back of the contract

12.45pm	Notes contract terms on spreadsheet Thinks about how to describe costs per run Types notes on bottom of schedule about cancelled episode. **Quantifies difference using computer functions in empty cell of spreadsheet** **Notes how difference arose at the bottom of the schedule** **In the Genersys section of spreadsheet calculates costs of episodes and puts against in transmitted rows** **eg 2 episodes @50k calculated 2*50 using keyboard** **Totals using autosum** Checks episode cost against contract schedule **Puts ?? against 120k cost for last three episodes** **Splits 3@ 40k into 2@ 40k and 1@ 14k ....** **Shifts ?? to cell beside 14k** In fee per episode deletes original note and inserts information from contract schedule Looks at print view Smartens spreadsheet
12.58pm	Moves to next sheet of spreadsheet *Lunch break...*

In the afternoon Sasha had a meeting with Sonjoy to progress some of the queries which have arisen in the course of the audit.

Exhibit 4 – Summary of observation notes; Sasha and Sonjoy discuss queries relating to programme accruals/year end stock

2.05pm	.......beginning of meeting
2.10pm	*This is summary of next 35 minutes. During meeting observer made notes of discussions on the nine schedules that Sasha talked to. Notes made on copies of those schedules.* ***Sasha takes out her programme accruals and stock schedules – for example see table 2 – and works systematically through her queries. She goes through her queries on each schedule, working systematically down the figures column starting with the accounts figures at the top of each schedule. Sonjoy looks at schedules as Sasha talks. In the case of differences between the accounts figures and the CODA records, Sonjoy explains clearly why there are accruals and stock figures and why the differences arise.*** *In the case of the differences between Genersys and the contract numbers and the CODA numbers, Sonjoy tells Sasha that she needs to talk to Aurora who really understands the records. [Sasha had been aware that one her problems in trying to do the Genersys based work was that Aurora was away and that she had to rely on schedules that had been printed out by someone who was not particularly familiar with the Genersys System.] Sasha questions and listens carefully and notes Sonjoy's comments on her schedules. Towards the end of discussion Sonjoy goes to his office to get a note for Sasha.*
2.45pm	

Exhibits 5 and 6 are taken direct from handwritten observation notes.

Exhibit 5 – Sasha and Sonjoy discuss commissioned programme accrual, S

2.10pm	**So, "There is an accrual because G....has not invoiced. This is not unusual. We are not going to chase for obvious cash flow reasons. G is slow."** S, "I've compared CODA and Genersys schedules. The Genersys records show all episodes bar one were transmitted in September." So, "Accounts *(CODA)* don't post by episode so we don't need the Genersys records. **Our liability is only for 9 episodes."** **S "It is a big invoice."** **So "We've got to find 400k from somewhere."**

Exhibit 6 – Sasha and Sonjoy discuss other items on Sasha's queries list

	S goes back to her o/s list ....
2.56pm	S "Can I have budget and cash flow for next year?" So "......the budget is not yet signed off by the Board......what we are going to do is to take the costs but we're going to rework the revenue......I will give to J when he is next around." S ".......also the agreement with K....?...." So "....Yep.......I'll give to Jonathan........."
2.58pm	**S "Debtors?"** **So "All debtors received by now"** **S "Have debtors gone down as a result of change in policy?"** **So "Difficult to say why"**

S listens and notes on her pad
Nods
Notes
Nods
So "......trade debtors take 30 days......In future there will be a decrease. T* now broadcasts in Scotland and Northern Ireland, so debtors will go down."
So chats about change.
S "....other debtors have increased....."
S takes out schedule from file
So explains
S listens and interjects "There are a couple of large prepayments......"
So explains
S and So share a joke

The tasks The first three extracts – exhibits 3 to 5 – relate to Sasha's test of the systems for transmitted programmes. We see Sasha creating and using her spreadsheet to test T*'s systems for accounting for the commissioned programme, S. In extract 1, Sasha sets up a spreadsheet using information extracted from copies of the company's records. The purpose of the spreadsheet is to reconcile the accrual as shown in the draft accounts with: (a) the numbers in the bookkeeping system (CODA); (b) those kept by the department responsible for monitoring the transmission of programmes (Genersys); and (c) those in the contracts with the suppliers. In exhibit 5, Sasha discusses the numbers in the spreadsheet with Sonjoy to clarify her understanding of the nature of the credit balance in the year end accounts and to confirm the reconciliation of the information in the accounts to that in other company records. Sonjoy was only able to provide a partial explanation. Sasha told me that she likes to set up Excel spreadsheets at a relatively early stage in her field work so "I can think about [what I have got] and decide what more I need." In exhibit 6, Sasha focuses part of the discussion of audit queries around figures from the draft accounts. Sonjoy follows the lead given by Sasha and Sasha listens carefully to the explanations given by Sonjoy and makes extensive notes.

The mathematics in the tasks Sasha, like Holly and Gary, read and understood financial records and extracted mathematical meanings from them (exhibit 3, lines 21-22, 27, 30-32, 42-45). In the course of setting up the spreadsheet she carried out 10 arithmetical calculations, always using the Excel programme functions, e.g., exhibit 3, lines 40-41. Also, to calculate the difference between £435,000 and £355,000, she used an empty cell in the spreadsheet and Excel functions.

The structure of Sasha's spreadsheet followed that already used for stock accruals. Its purpose was to compare the accrual (a number) in the draft accounts to the numbers generated by the CODA and Genersys systems and the contract with the supplier. The kernel of the test of T*'s systems comprised setting up a numerical comparison of the numbers produced by company records. It is, amongst other things, a mathematical model and a proof of particular numbers. Sasha both set up the model and used it. She highlighted the figures in the copies of the company records with a marker pen and then recorded them in the spreadsheet in a columnar format enabling easy comparison and she checked the validity of the figures to be used to the contract (lines 30-32). The contract numbers did not agree with the other totals, so the difference was calculated and explained in a footnote.

In exhibit 4, we see Sasha discussing the results of her draft spreadsheets with Sonjoy. Some of this activity was mathematical as it involved asking for explanations of differences. The activity in exhibit 5 on a first judgement looks mathematical but on further examination is seen not to be – see below. On the other hand the activity in exhibit 6 looks completely non-mathematical but that is not quite the case. When Sasha notes that debtors have gone down (line 11) and other debtors have gone up (line 22), she is setting out the result of two mathematical calculations as she has compared the current period end balances with those of the prior period. It is also arguable that Sonjoy in talking about his expectation that the debtors will decline in the future engaged in an activity that involved some mathematics.

A closer look at the transcripts hint at other types of mathematical activity related to Sasha's audit work. It suggests that Sasha used a stratified sample to test the recording systems for programmes. She also asked for the company budget and cashflows so she could review them.

Numerical activity that is not mathematical These extracts are notable for much activity that uses numbers/mathematical operations that are not mathematical in character. In line 14, exhibit 3, Sasha is just copying information from one document to another. Sasha uses the spreadsheet model for non-mathematical purposes; to summarize the information she has got and to highlight what she needs (though in this case the totals for each system guided her judgement as to whether she needed more information); and as audit evidence. The purpose of the brackets in the calculation of the CODA balances in table 2 (see also lines 21- 22, exhibit 3) is not to indicate the mathematical operation of subtraction but to indicate that the programmes have not been invoiced by the programme maker even though they had been transmitted. In exhibit 4 the modelling spreadsheets are used as part of a detailed agenda for Sasha's discussions with Sonjoy. The discussion in exhibit 5 is not mathematical activity even though reference is made to several quantities; Sonjoy is merely explaining why the creditor has not been paid and stating that T*'s liability was only for nine programmes.

5.2.5 Leonora, the administrative assistant, arranges a secretaries' meeting

On the morning of the observation, Leonora undertook a large number of different administrative tasks. She had an organised and systematic approach to her work. In particular, she had a large number of personal procedures and practised routines which enabled her to carry out her work effectively. Leonora prepared and submitted to the firm's stationery supplier a substantial stationery order. This involved judgement and a substantial amount of simple arithmetic. This episode is discussed in more detail in chapter 6 to discuss a use of numeric codes which does not have a mathematical character. Here, I have included an outline of Leonora's approach to arranging a secretaries' meeting as this was typical of her systematic and logical approach to her work.

Exhibit 7 – Extract from note based upon my observation notes; Leonora arranges a meeting.

Arranging the secretaries' meeting Leonora adopted a methodical procedure for determining the date for the secretaries' meeting. The task was to find the time that would suit David Ward, who was the managing partner of the business unit, and most of the secretaries	

Leonora had prior to the day's observation e-mailed all secretaries requesting their availability
By the day of observation most had replied
On day of observation she printed out replies

> **Set up Excel list with those names and availability**
> Printed it
> **Ticked off names on list against distribution list in the Outlook address system**
> **Added missing names to bottom of Excel list**
> **Rang round to get availability or best estimate of availability of those who had not replied**
> **Got David Ward's availability from his secretary**
> **Checked David Ward's availability - four possible dates - against Excel list**
> **Chose 19th March as best**
> **Checks conclusion by ticking off 19th availability on list. Almost all secretaries can attend.**
> **Decides on 19th**
> Advises all by e-mail

(This note was made three days after the observation.)

Mathematics in the task This was a successful exercise of practical mathematical logic using a process/model that was 'fit for purpose.' It is rigorous except in one respect. Leonora could not wait to make a decision until she had details of everyone's availability; she went ahead with her decision-making process because she had answers for almost all the secretaries.

5.3 Concluding Remarks

The evidence and discussion in this chapter is mainly used to underpin the definition of *workplace mathematics* developed in the next chapter.

Before turning to this, I highlight two skills that all my participants used as they worked: reading and understanding texts, and using the firm's referencing system for ordering working papers, as they are of relevance to my development of the concept of *workplace mathematics* in the next chapter.

Throughout this chapter and the rest of this thesis, participants are reported to be reading and understanding texts, many of which make substantial use of mathematics. This activity is fundamental to auditing in a literate society and is a key prerequisite to most of what the participants did. How do I know that they read texts? Because I watched them do so and sometimes they read extracts aloud. How do I know that they understood them? Part is supposition on my part, but mostly through what I observed them do and, in particular, how they acted when understanding and meaning were not seen as obvious and are judged to be important. Later in the thesis I show how participants extracted/constructed meanings from texts. This reading for understanding almost certainly does involve the exercise of some mathematical skills. However, my method of data collection and analysis focusing as it does on action 'out of the head' does not enable an in depth analysis of this skill. Nevertheless proficiency in reading financial information for understanding was an essential skill that was required by all my participants as they worked.

The second key skill is more specialised. All documents in the firm's client working papers are referenced using an alpha-numeric code. Reference numbers are attached to all documents and to some specific information in the documents. The ordering properties of the alphabet and the numbers encoded into the referencing system determine the order in which documents are filed and thus enable them and information therein to be located easily. The referencing system enables a reader to trace the figures in accounts through a series of schedules to the figures produced by the accounting system. In this chapter and throughout this thesis, I record participants both referencing and filing new working papers and using the firm's referencing system to locate papers or information in papers; their effective and efficient of the system depends upon both their knowledge of and their effective and efficient use of the ordering property of numbers and the alphabet incorporated into the system.

6 *Workplace mathematics:* recognition and definition

6.1 Introduction

In this chapter I develop a definition of *workplace mathematics* using data from my phase I fieldwork, Nunes et al.'s definition of *street mathematics,* and a number of theoretical considerations, including Luhmann's concepts of observation, complexity and communication[1]. I then discuss the characteristics of three particular categories of *workplace mathematics* which were used by my participants. However I begin by discussing how I identify participants engaging with mathematics as they work.

6.2 Recognising participants using mathematics

In chapter 2, I have used Luhmann's systems theory to distinguish between the mathematics in use in the discipline of mathematics and that in use outside the discipline. Use outside the discipline includes use in ordinary work practices. This distinction changes key questions from 'What is mathematics?' and 'How do I recognise mathematics?' to 'How do I recognise participants using mathematics as they work?' and 'What are the characteristics of *workplace mathematics?*' Before I explore the second question, I consider the first briefly and derive a definition of *workplace mathematics.*

How do I recognise participants using mathematics as they work? In reformulating the question for consideration I have chosen the word 'using' to convey participants' active use of mathematics in the course of doing their work as my thesis is about how individuals use mathematics in the workplace. The question is a 'how' question not a 'what' question. Any answer has to deal with the empirical, 'How do I recognise....?' and the theoretical, 'What qualifies as mathematics?' I deal with the empirical first.

This question, ' How do I recognise...?', belongs to the system of academic research. My perspective is that of mathematics education research. I search through my observation data to find participants using mathematics. As I reviewed my field notes, I did not observe the discipline of mathematics in action but observed individuals working as auditors and an administrative assistant. I chose to distinguish from the work practices observed (i.e., the background), activity where participants appear to be using mathematics. I then analysed these chosen practices to distinguish and describe how participants use mathematics. Through the process I recognized mathematics and the use of mathematics within the activities of work. Implicit in my observations are presumptions about what constitutes 'mathematics' and 'using mathematics'. These need to be made explicit. In identifying mathematics in use I relied heavily upon my tacit knowledge derived from my past experiences; I have a mathematics degree and I worked for over 20 years as an accountant in an international accounting firm and for several years as a volunteer in the mathematics department of a secondary school.

In some cases mathematical activity was obvious, e.g., where calculations were performed but in other cases it was not and only emerged upon reanalysis, e.g., recognising that Holly's deduction that the payment of a dividend might be illegal was

[1] Discussed in section 2.4

a deduction that could only have been derived from the numbers and structure of a particular company's accounts. The results of this process of observation are discussed throughout this thesis. Those described in the previous chapter are used to draw out some of the characteristics of *workplace mathematics* in order to develop the definition.

The theoretical answer to the how question is, of course, different. Mathematics certainly includes the branches of knowledge, which are usually classed under the name of 'mathematical knowledge', e.g., arithmetic, algebra, statistics and analysis. It it is more; mathematical processes, including inductive and deductive reasoning, are also part of mathematics.

> "[T]he one word "mathematics" is habitually used in two senses, and so, ..., I have distinguished between "mathematics", the methods [meaning 'processes'] used to discover certain truths, and "Mathematics", the truths discovered" (Jourdain, 1960, p.7).

These views written in the early part of the 20th century are not dissimilar to those of Wittgenstein discussed in chapter 2, even though Jourdain was clearly a Platonist. Jourdain saw mathematics as part of a two way street. Jourdain considers that mathematics emerged (and continues to emerge) as abstract observations and deductions from practical problems, while mathematical results and techniques were (and continue to be) applied to provide or assist in providing solutions to practical problems.

This analysis sets a boundary almost identical to that derived from Luhmann's systems theory and enables a theoretical answer to what ought to qualify as *workplace mathematics*. For the definition of *workplace mathematics* to be credible, it is important that that the activity in the ordinary workplace that qualifies as *workplace mathematics* is consistent with the mathematics of the discipline. This suggests that to qualify as *workplace mathematics* practice should involve the use of mathematical objects, results, structures and/or methods/processes and should be used in a way consistent (or intended to be consistent) with the rules of mathematics. By mathematical concepts[1] I mean the objects, results, structures and/or methods and processes of the discipline of mathematics, or to use Jourdain's formulation – the mathematical truths discovered and the methods used to discover them.

Generally usage outside the discipline is not as rigorous as that within the discipline.

6.3 *Workplace mathematics* – developing a definition

Many approaches to theorizing about using mathematics in the workplace are possible. I chose to ground my definition of *workplace mathematics* in the practices observed. The mathematical activities observed are summarized below. As I did not want to reinvent the wheel I decided to adapt Nunes et al.'s (1993) definition of *street mathematics*. Directly as a result of adapting the definition, it appears that what can count as mathematical activity goes beyond the limits set for *street mathematics*. I, therefore, set new conditions activity should comply with if it is to be regarded as *workplace mathematics* and so arrive at the definition. In the conclusion to this chapter, I list a number of examples of apparent mathematical activity which are not

[1] Definition of 'mathematical concepts'

workplace mathematics and discuss possible valid and invalid use of *workplace mathematics* in work practices similar to those observed.

Street mathematics / workplace mathematics

I use my deconstruction of the work practices observed in Phase I and discussed in chapter 5 to develop a definition for *workplace mathematics* in the same way Nunes, Schliemann, and Carraher (1993) searched for and found *street mathematics.* The deconstruction has been used to surface the mathematical activity embedded in the work observed. Much of this activity is described and summarized below

In chapter 5, I identified a number of different mathematical activities/practices used by the participants. Holly read and reviewed C1's draft accounts; balance sheets and profit and loss accounts are mathematical models. She checked the mathematical integrity of the accounts by checking numbers were consistent throughout and the arithmetic accuracy of all totals and subtotals. She also used logical reasoning and inferred from the mathematical structure of the accounts that a proposed dividend was funded out of intercompany debts. Gary read and understood the numbers in the remuneration schedule. He checked all totals in the remuneration schedule. He reconciled (proved) the salaries of the directors, other than the managing director, to copies of the accounting records in the audit working papers; in two cases this involved deducting pension contributions paid from the salary figures in the remuneration schedule. Gary used a series of exploratory calculations to help him work out how the managing director's salary in the accounts was made up and how it agreed to his contractual entitlement. He then set out a proof of the charge in his working papers, as follows:

$$£240{,}000*8/12 + (0.8*£100{,}000)*8/12,$$

Gary's proof reflected the way he built up an understanding of the position. Sasha, like Holly and Gary, read and understood numbers in financial records. She highlighted figures she deemed to be significant. She, also, created an Excel spreadsheet model to make numerical comparisons between the accrual for the amount due to be paid in respect of a series of transmitted programmes against information generated by other company records. In the course of constructing the comparisons, she performed a number of straight forward arithmetic calculations. Sasha and Sonjoy in their meeting compared and contrasted current and prior year debtor balances. Leonora set up an array tabulating secretaries' availability, in effect a truth table, and used logical reasoning to determine the date for a meeting. Aside from possibly the checking of the arithmetic integrity of financial records, all of these mathematical activities/practices were an integral part of a non-mathematical activity, such as reviewing a set of accounts, checking the directors' remuneration note in draft financial accounts, testing a firm's systems or determining the date of a meeting. It is also to be noted that Holly, Gary and Sasha always made calculations using either a calculator or the Excel functions.

As my participants worked they engaged in the following mathematical activities:

- arithmetic,
- reading and understanding mathematical models and financial information,
- checking the mathematical integrity of financial records,
- reconciling (proving) numbers – in this case reconciling accounts numbers to numbers in company records,

- making exploratory calculations to aid understanding,
- creating and using spreadsheets to enable numerical comparisons of numbers derived from accounting records,
- comparing and contrasting 'like' numbers,
- constructing a truth table, and
- making deductive inferences from information in models.

These mathematical activities/practices used in the course of the everyday work are indicative of part of a possible working definition for ***workplace mathematics,*** being *'mathematical activity carried on within the workplace.'*

This is analogous to the working definition of *street mathematics* used by Nunes et al. (1993), *street mathematics* being "mathematical activity that is learned and carried out outside school." Following detailed analysis of the observations in their empirical studies, Nunes et al. sought to characterize mathematical activity more clearly. They consider it as generally characterised by deductive reasoning from postulates and as not concerned with observation, experimentation or causation. It is concerned with the exploration of relationships between representations, not with the relationship between events. It is about making inferences from the representations (i.e., mathematical models). The mathematical activity of *street mathematics* is embedded in another activity that gives meaning to the situation. The street traders, carpenters and fishermen observed engaged in mathematical activity in the course of other activities; they mathematized situations through model building. Something as simple as the calculation of the price for three coconuts constitutes mathematical activity; as does estimating how many different standard measures of wood are necessary to make a bed of given dimensions. The use of specific information about the event within the model allowed the participants to control for meaning and reasonableness of their answers. Nunes et al. take a narrow view of what constitutes mathematical activity; it is only the drawing of conclusions from the models and not the model building (Nunes, 1993, p.130).

Although analogous to *street mathematics,* I propose that my working definition of *workplace mathematics,* which is derived from the ordinary work practices observed, has a wider meaning. I propose that it should not exclude mathematics learnt at school or college as my participants were all well educated, and primary and secondary education is universal in the UK. Furthermore it should include specifically the reading and understanding of texts with embedded mathematical content and the building as well as the using of mathematical models created to represent an aspect of activity or a hypothetical situation. These differences are not problematic and possibly arise from the fact that *street mathematics* is concerned primarily with substantially non-literate work cultures. This suggests a minor extension to the working definition is needed: '***workplace mathematics*** *is mathematical activity carried on within the workplace as part of ordinary work practices.'*

However whether *workplace mathematics,* like *street mathematics,* should be concerned solely with the exploration of relationships between representations of aspects of events and hypothetical situations, and not (or only indirectly?) with the relationship between events and situations, is slightly more problematic. It seems to me that the mathematical activities which my participants engaged in sometimes involve manipulating 'real' world objects or texts mathematically. For example: (a) Gary and Holly used the firm's alpha-numeric referencing system to locate both working papers and numbers therein; (b) Gary also used it to reference, order and file new papers; and (c) Sasha totalled information about particular contracts from different systems to determine the completeness of her on-going investigations of particular balances in each system. Using the principle of action in context as a way of viewing cognition, a key tenet of the situated cognition paradigm, I propose to extend radically the definition of *workplace mathematics* to include implementation involving action in the world, provided the practices clearly involve using the rules or methods of mathematics.

Constraining the definition

Within the context of Nunes et al.'s definition, it is no longer obvious what is and is not mathematical activity. I therefore propose that for activity to qualify as *workplace mathematics* should be constrained in two ways, resulting in the following definition:

***Workplace mathematics** is mathematical activity carried on within the workplace as part of ordinary work practices. Two defining principles must be present for a mathematical activity to be recognised as workplace mathematics;*

- *part or all of the work practice (system) must incorporate at least one mathematical concept*[1] *which is used in accordance with the rules of mathematics; and*
- *usage occurs when a participant acts in a way that is consistent with the mathematical rules that apply to the concept/s such that either meanings are given to information, or actions and/or events in the 'real' world are facilitated.*

All the mathematical activity described above, including in particular those encompassing action, i.e., (a), (b) and (c) in the last paragraph of the previous subsection, fall within the constraints. Thus the definition is sufficiently broad to encompass activity that 'looks' mathematical without mathematizing everything numerical – see section 6.5.

6.4 Major categories of *workplace mathematics* observed

6.4.1 Introduction

Now I turn to consider three broad categories of *workplace mathematics* and their characteristics: mathematical activity in modelling; partially following Luhmann, mathematical activity as part of the devices used to organise and simplify complexity; and mathematical activity related to the uses to which money is put. Some number related activity is obviously *workplace mathematics*, e.g., checking the integrity and accuracy of calculations, reconciling accounts figures or making logical deductions from a mathematical model. Others less so.

Mathematical modelling of 'real' world problems/situations is a widely recognised use of mathematics but what part of modelling is mathematics is, as is apparent from the discussion above, contested. The discussion below sets out where I propose to draw the line for this thesis.

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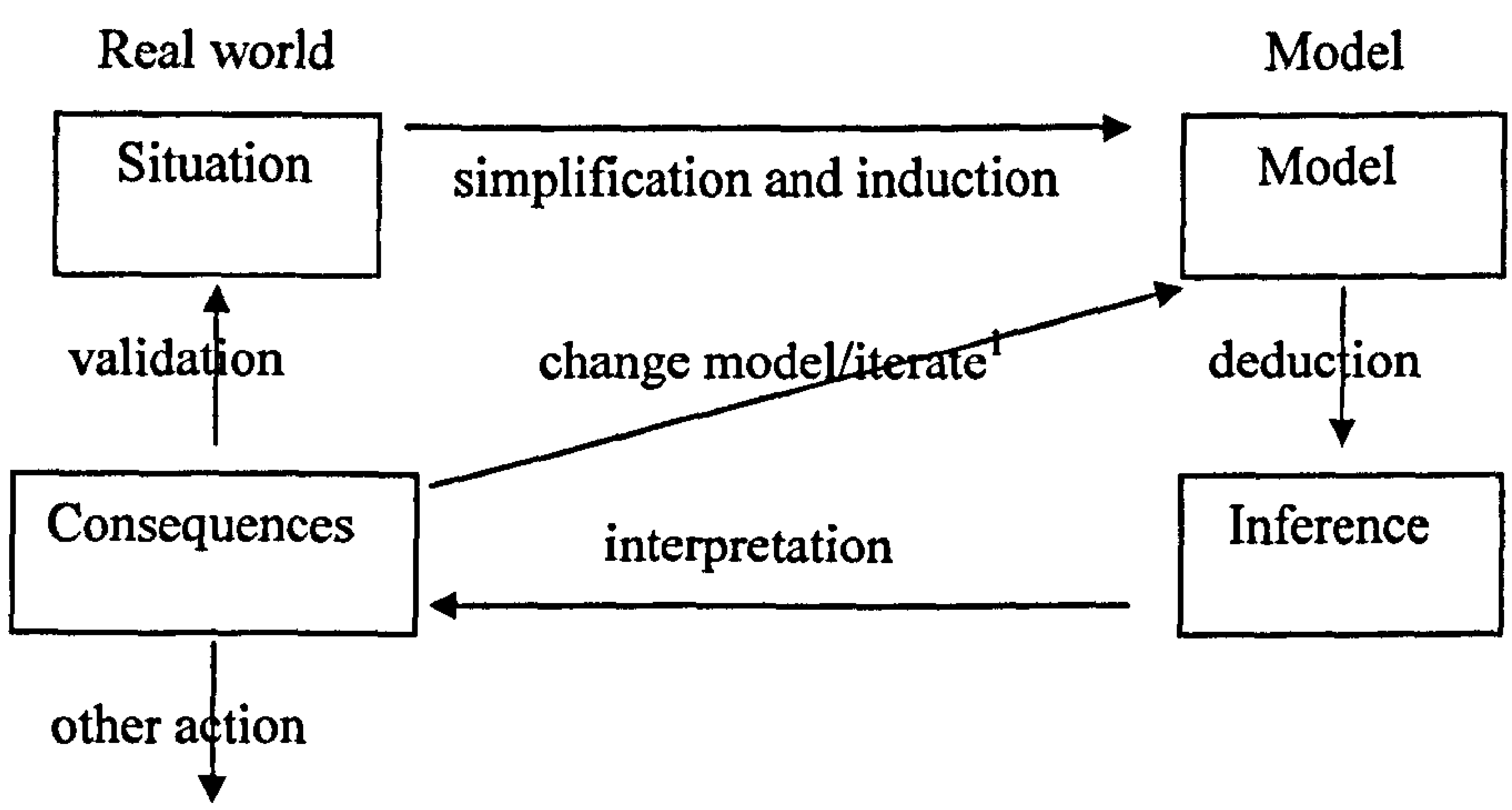
[1] See section 6.2 for definition

My second category, using mathematics to organize complexity, is fleshed out using ideas from Luhmann's systems theory and consequently is idiosyncratic. Activities in this category are one of the key ways my participants used mathematics. Finally, I refer briefly to the role money plays in my participants' work.

6.4.2 Mathematical modelling in practice

Davis and Hersh (1983, p.78) following Aris (1978) describe a mathematical model as being an abstract representation designed to correspond in some way to a situation, e.g., a physical, biological, psychological, economic or conceptual entity. They also consider that it must be a complete set of consistent mathematical equations or structures that corresponds in some way to the entity. Alternatively I could have used Brown's (1999) analysis. Here the key issue is to consider how mathematics hooks into the working world with which I am concerned. It takes on a representational character. It hooks into the world (the non-mathematical realm) by providing representations in the form of structurally acceptable mathematical models of properties of objects (tangible and intangible). This representative view of applied mathematics can be accommodated by Platonists, constructivists and structuralists.

Modelling, as defined above, is seen as involving simplification, part of the purpose of which is to represent the relationship between salient features. Generally, situations/problems in the real world cannot be translated directly into a mathematical equations/structures. It is often necessary to simplify the situation/problem to produce a set of equations/structures which concentrate on the essential features and their relationship with each other and ignore the rest. Interpretation of mathematical deductions from such models can often result in action in the world. Frequently, however, the simplification is so drastic that the mathematical deductions derived from the model, when interpreted in real world contexts do not produce sufficiently reliable predictions or representations of situations or possible situations. In these cases the model requires amendment or the results need to be interpreted with caution. The modelling process described emphasises the separation of the 'real' world and the model as is illustrated by this diagram – an adaptation of similar diagrams seen in A-level and first year undergraduate mathematics textbooks:



[1] Suggested partly by observations in the field

This diagram divides model into three: (a) the 'real' world, (b) the model and (c) the processes whereby a 'real' world situation is transformed into a model and vice versa. I propose that both (b) and (c) should be, generally, be regarded as *workplace mathematics*. Nunes et al. chose only (c) to be *street mathematics*. My choice is based upon the assumption that much mathematics is used as models are built, assessed and modified. For evidence see section 8.5 in particular.

Davis and Hersch (1983, p78) in *The Mathematical Experience* write:

> "Some of the purposes for which models are constructed are (1) to obtain answers about what will[1] happen in the physical world (2) to influence further experimentation or observation (3) to foster conceptual progress and understanding (4) to assist the axiomatization of the physical situation (5) to foster mathematics and the art of making mathematical models."

I could have used many other lists. I chose this list as the first three categories cover much of the modelling that is *workplace mathematics* and it recognises that mathematical models are used for many other purposes both within and without the discipline of mathematics. Given the nature of the work that my participants engaged in, i.e., assessing what has happened in the past or what still subsists, (1) above needs to be extended to cover, not only what will happen, but what has happened and what is (at the time of modelling) and what might happen. My participants used mathematical models for the following purposes:

- reviewing models to ensure that they are reasonable given particular facts and circumstances (1) – this was the main objective underlying Holly's review of the accounts and Sasha asked for the company's budget and cash flow for the current year so she could review them (this latter activity was not observed);
- assisting in developing theories of what had happened in the real world (3) – Holly used the results of a draft set of accounts to deduce that a dividend paid might be illegal;
- influencing further investigation (2) – Gary modelled the recharge of the MD's salary from its parent company and used this model to establish via invoiced amounts that the salary had in fact been recharged;
- representing events, e.g., the outcome of commercial transactions (1) – Sasha set up the spreadsheet model to compare the amount shown as owing by the company in the draft accounts with the amounts shown as payable by company systems; and
- confirming contractual prices – Sasha in her spreadsheet model used the standard pricing model to price (and hence check the prices payable for) television episodes transmitted.

All the observed activities, which involve using and/or creating mathematical models, to a greater or lesser extent described fall within the definition of *workplace mathematics*. However mathematical modelling, as defined here, is not wide enough to encompass the diversity of mathematical activity that I encountered in my phase I data. This brings me to a distinctive category of usage – using mathematics to organize complexity.

[1] It would have been preferable to have used 'might or will' instead of 'will'.

6.4.3 Using mathematics to organize complexity

Within some traditional mathematical modelling paradigms, models are seen as abstract representations that are used as devices with which to reason. Some of the *workplace mathematics* in the work practices observed do not quite fit this paradigm:

- Holly and Gary used the firm's alpha-numeric referencing system actually to file and find working papers and to highlight and find specific numbers therein; and
- Gary and Sasha not only used models to represent information and events, they used them to structure the collection of information and the extraction of meaning from it.

Holly and Gary use the ordering properties of numbers and the alphabet to file working papers and to find information in those papers. Gary and Sasha used mathematical calculations to aid the understanding of information and to determine additional information that was needed to complete tasks in hand. In each of the cases described here the participants used *workplace mathematics* as defined. They used it to facilitate events in or the understanding of situations in the 'real' world. *Workplace mathematics* was used as a device to guide observation (selection) and action, and consequentially sometimes to provide structure to communications. It was used to reduce noise in communication. Thus, following Luhmann, it sometimes operates as one of the codes participants use to organise and simplify complexity.

6.4.4 The role of money

Money is a major driver of *workplace mathematics* because its quantitative properties, which are inextricably linked with the quantitative properties of number and the four arithmetic operations, are in the modern economy divorced from its form. This is particularly true for my participants as their work was directly or indirectly concerned with accounting for monetary transactions and values. It is therefore worthwhile summarizing the function money plays in our lives (closely following Begg, Fischer and Dornbusch, 2000, p.375-377). The way in which my participants whose activities are described in chapter 5 encountered money are described briefly in brackets after each function is described. Money is any generally accepted means of payment for goods, services and settlement of debt and hence is a medium of exchange (placing purchase order for stationery). It comprises, amongst other things, coins, notes, cheques, bankers' drafts and bank deposits. Money has other functions:

- it is a unit of account for accounts (reviewing accounts and remuneration schedule) and for valuing assets and liabilities (assessing reasonableness of amounts shown in balance sheets when reviewing accounts);
- it is a unit of account for calculating prices (pricing television programmes using information from contracts and calculating the quantum of the recharge for the managing director's salary);
- it is a store of value enabling purchases or distribution of profits to shareholders to be made in the future (funding of dividend); and
- it serves as a standard of deferred payment, e.g., loan repayments (checking amounts due to be paid for programmes transmitted).

Money is thus a major driver of my participants' work and hence their use of *workplace mathematics*.

The three major drivers of usage described are not mutually exclusive. There is overlap between them particularly between money and modelling. The categorization is, however, useful as it helps explain the extent of *workplace mathematics* encountered in phase II.

6.5 But all is not included

Workplace mathematics does not include all work practices involving numbers. Sasha and Sonjoy in their meeting focused a large part of their discussion around the numbers in the draft accounts and those produced by T*'s financial systems but did not necessarily engage in mathematical activity. Leonora's use of the codes for the stationery items was non mathematical. She typed the alpha-numeric code of an item from the stationery order request into the computerised ordering system. She always checked the description of the goods which appeared on the computer screen against the description on the request. The description always took precedence. Thus Leonora did not use the numeric properties of the codes as she worked. Other activities involving the numbers which are not within the definition of *workplace mathematics* are discussed at the end of subsection 5.2.4 on Sasha[1]. Arguably aspects of these practices might also be examples of *workplace mathematics.* However, I am hesitant to argue strongly for their inclusion as my concern is not the mathematization of accounting and auditing and other work disciplines but to discover how the participants actively engage in mathematical activity as they undertake their work within those disciplines.

Workplace mathematics is tightly connected to mathematics by the way it is used; valid, correct or legitimate use should not violate the rules of mathematics or to put it more strongly should follow the rules of mathematics. It does not solely consist of using it correctly. Making errors and/or failing to find a satisfactory solution as one works is part of the practice of mathematics both within and without the discipline of mathematics. Using invalid or incorrect methods and results may be part of *workplace mathematics*. On the other hand where they are used to justify or evidence real or possible situations in the commercial world may not be. Whether the practice is that of *workplace mathematics* or rhetoric alone is probably a matter of motivation – for further discussion see chapter 9.

My participants did not set out to do mathematics as such. They set out to perform particular tasks and in the course of carrying out those tasks engaged in mathematical activity. In following five chapters I analyse in depth the data collected through observation, particularly that from phase II, to illustrate nature and extent of *workplace mathematics* used by my participants as they worked.

[1] See part of subsection on Sasha starting '**Numerical activity that is not mathematical**'.

7 Presenting financial information in tables: introducing *workplace mathematics* in action

7.1 Introduction

In this chapter, the first of five dealing mostly with data collected in phase II, I show how *workplace mathematics* is used to order and present information using two episodes. I use Hutchins's theory of distributed cognition to analyse the way in which participants are influenced by routines, available resources and the settings in which they work. I also describe in depth the participants' mathematical activities that were embedded in the work activities observed. After discussing the first episode, I provide in section 7.4 an important interim conclusion about those activities. In section 7.6, I compare and contrast the two episodes and briefly compare them with some phase I practices described in chapter 5. I then conclude with a discussion centred mainly on the usage of *workplace mathematics*. I start by discussing the auditing of specific bad debt provisions with particular reference to the audit team's practices. This is to provide background and context for many of the examples used in this and subsequent chapters.

To aid reading and conciseness, I have adopted a number of conventions in this chapter:

- all coloured entries in exhibits – other than those highlighted in yellow – are my additions to working papers;
- numbers in **red** indicate the order in which the participant entered information into a table;
- numbers in **purple** indicate the position of a column in a table; and
- numbers/letters in **blue** highlight and number mathematical operations used by a participant.

7.2 Loan loss reviews

7.2.1 Assessing specific bad debt provisions

Loans to customers are a substantial asset in a bank's balance sheet. An auditor's job is to assess whether or not the values attributed to the loans in the balance sheet are calculated in accordance with generally accepted accounting standards. For the 2003 year end audit, the Bank's audit team reviewed the large loan exposures (defined to be those individually over £30 million) considered by the Bank to be wholly or partly irrecoverable or at risk plus a few others. For each customer selected, the audit team assessed whether or not the Bank's specific bad debt provision (including a nil provision) was reasonable given the facts and circumstances of the particular case. The key assessments applied here were whether or not the debt was considered to be either wholly or partially irrecoverable and if it was, that the provision against the debt was the amount considered to be irrecoverable **at the year end**. The auditor's job is to assess the reasonableness of the Bank's provisions, not to set them. The review is effectively a review of a review.

7.2.2 Doing reviews

The review was carried out by a team of three: Tom, the audit partner; Cliff, the senior manager; and Eric, the audit in charge, who was a one year qualified accountant and was working on the audit for the second year running.

Tom had decided that Eric should review between 30 and 40 of the Bank's large exposures. By the time I observed Eric he had carried out almost 30 (as far as the review stage) so he was well practised. The review process was broadly as follows:

- Eric drafted a working paper for each loan and constructed a file to evidence each paper.
- Cliff and Tom reviewed Eric's work separately. After Eric had briefed them on a batch of loans, they reviewed the papers and files, and then had a follow up meeting with Eric to go through their review points.
- An audit opinion for each loan emerged during the process and was documented.
- In any case where questions about the Bank's provision arose, the questions were discussed with the client and resolved.
- Towards the end of the audit, Eric tidied up the files and Cliff conducted a closing review. And
- The team reported the results of its review to the Bank's Audit Committee.

7.2.3 The Wallaby loan loss review – a typical review

In March 2003, Wallaby underwent a major reconstruction. It converted a large part of its debt to equity and wrote off its losses against the resulting share premium account. Loan creditors, including the Bank, received equity and debt in lieu of most of their outstanding loans. The Bank formally wrote off a large part of its original loans, having already provided against them. The Bank continued to provide Wallaby with other finance, mostly secured. In late 2003, the team reviewed and accepted the Bank's assessment that no provision was needed against its exposure to the reconstructed company.

a) Eric creates the working paper and file

Eric compiled the working paper using his pre-existing knowledge, information in prior period audit files, information from the Bank's records and staff, Wallaby's accounts and other public information. Before starting, Eric assembled some papers – a computer listing of the loans to Wallaby as at the date of the review (the CLM), the Bank's latest review of its exposure to Wallaby, the audit half year working paper and some press articles. As Eric worked he downloaded the Wallaby's 2003 Q1 and H1 results (first quarter and half year results) from the internet. Eric drafted the working paper on his PC starting with a blank proforma working paper (appendices 1 and 2) and created a 'good' working draft using a methodical process, which comprised:

- filling in housekeeping details, such as the Bank's credit grading for Wallaby, the provisioning position and a few brief details of the previous review;
- creating a table analysing, summarizing and totalling the approved facilities and amounts due to the Bank at the date of the review (exhibit 3). This was derived from the CLM and the Bank's review of operations only;

- providing an update on the latest credit position. This was mostly derived from the Bank's review of operations;
- making an assessment of how the Bank might be repaid. This was mostly based upon information extracted from Wallaby's Q1 and H1 results;
- concluding that the Bank's provisioning position was reasonable. Eric made this assessment before the H1 results were published. His conclusion did not alter as a result of his review of the H1 results; and
- minor redrafting after telephone discussions with the client to sort out several minor queries.

Eric's working paper is a working paper, not a report. Information which became redundant because new information became available was often neither deleted nor updated. The text is essentially notes interspersed with comments leading to an audit conclusion; it is not carefully crafted text intended for publication. It was a working text for the team and is a record for the file.

b) The review process

This was as described above except in one key respect. Tom had a private informal discussion with Eric after his review, in which he explained to Eric how he should carry out analytical reviews of accounts to assess whether or not a company is likely to able to repay its debts. Following that meeting Eric wrote up notes of the discussion (appendix 4) and then on 3rd December wrote a paper re-analysing the H1 results (appendix 5), focusing on the quantum of the total debt outstanding, current and hypothetical prospective cash flows, and the likelihood of the debt being repaid out of future operating cash flows. For discussion see section 10.7.

c) The audit record

A copy of the final working paper was filed in the 2003 audit file together with the supporting evidence. The file is an essential part of the audit – it contains the evidence on which the audit decision was based.

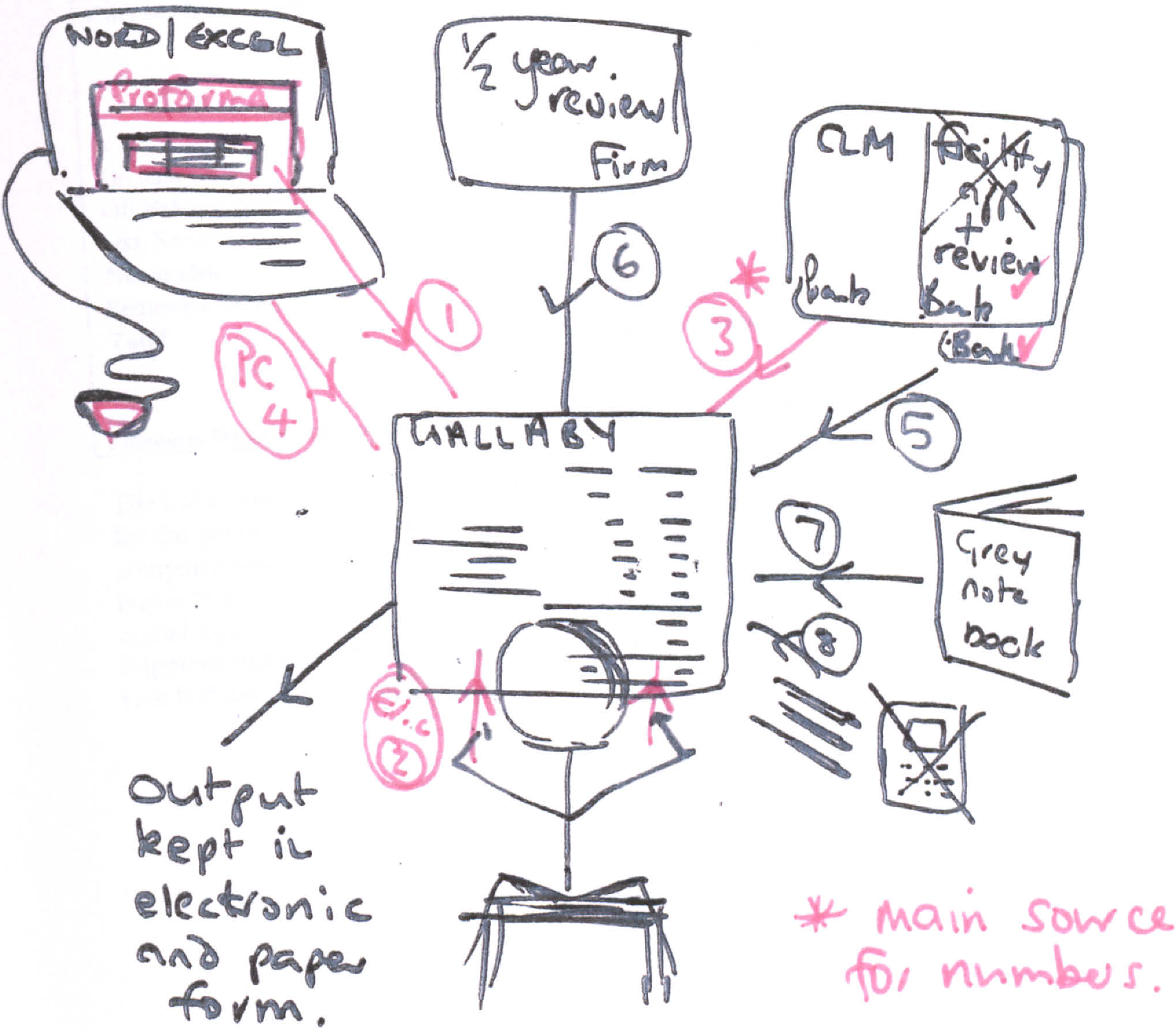
7.3 Wallaby – summarizing the loan balances

7.3.1 Introduction

In this section, I use Eric's proforma Excel spreadsheet (exhibit 1), my observation notes (exhibits 2 and 4), and Eric's table of the Bank's loans to Wallaby (exhibit 3). The episodes under analysis are the construction of the table (exhibit 3) using the proforma (exhibit 1) and Eric's and Cliff's use of the table. The exhibits need to be read in conjunction with the discussions that follow exhibits 1-3 and exhibit 4, since I decided not to annotate my observation notes extensively in order to preserve them as contemporaneous records of my observations.

Figure 1

Schematic diagram of Eric working at his desk constructing the summary of the Wallaby loan position



○ all necessary to table

○ supplementary information / tool

→ input ← output

7.3.2 Eric at work

Before Eric started drafting his working paper on the Wallaby loans he set out on his desk everything he needed to construct a good first draft – see figure 1. Eric's first substantive subtask (described in exhibit 2) was to compile the table summarizing the quantum of the authorized facilities and the loans due to the Bank from Wallaby (exhibit 3). He started with this proforma Excel spreadsheet:

Exhibit 1 – Excel proforma; from Eric's Word proforma working paper

Current facility position	**Facilities £'000**	**Outstandings £'000**
x		
On balance sheet		
Off Balance sheet		
less Security		
Net at risk	-	-
Settlement facilities		-
Total	-	-

Footnote: Outlines of Excel formula cells do not show up except when working on screen.

The information for the table came from the complex limits monitoring report (CLM) for the group (3)[1] and the Bank's current review of operations (5). The CLM is a daily computer listing of the amounts due to the Bank as of the previous day's close of business and the corresponding authorized lending limits. The CLM and Eric's knowledge of the Bank's abbreviations were the main source for the input to the table. Supplementary information and clarification were obtained from the other documents Eric had assembled (6 & 7).

[1] Numbers refer to numbers on figure 1

Exhibit 2 – Extract from my observation notes; Eric constructs the Wallaby summary table

Headnotes
The text in italics summarizes several actions.
Text in [ ] added to aid reading.
** Row numbers from exhibit 3 inserted into field notes when the notes were first read for sense.
Eric's grey notebook was an A4 note book in which he kept notes of queries, telephone conversations and short meetings with client.

E classifies the authorized limits on the CLM, writing his classification on the CLM in blue biro.
E refers to the 'Bonding Facilities' section of the Bank's review, which he highlights with an orange highlighter.
Marks up bonding facilities as such on the CLM in blue biro.

8.50am Goes to the empty Excel spreadsheet in the 'Current facility position' box in the working paper [exhibit 1] *and inserts the text and numbers (other than those in the brackets) into the rows numbered 1-5** on exhibit 3 using information from the CLM.* [The entry in **column 2**, row 5 was altered subsequently – lines 25-26.]
As numbers are inserted into table they are ticked off on CLM in pencil.
E does some reformatting of table.
Enters information in row numbered 6.

Looks at the working paper relating to half year review (i.e. previous review) and page 2 of the bank review[1].
Sees [perhaps ' focuses' would be a better word] one item o/s on CLM and writes in pencil 'fx[2] lines?' against item on the CLM.
Refers to Bank review and notes in grey notebook.
Goes to the facilities table in the working paper and increases the total in **column 2**, row 5 by adding a number in the formula cell of the spreadsheet. [Process referred to in more depth in discussion of computational practices.]
E checks the numbers in the previous working paper against the numbers in the table on screen; he has his finger on the numbers in the working paper as he checks against those on screen. In this process he also refers to the CLM.
[It is to be noted that the numbers in the two working papers are slightly different and have been subtotalled slightly differently.]
Moves row 4 above row 1.
Types text in brackets in rows 1 and 2 from highlighted information in the Bank review.
Inserts row 7 after row 1 and types text into row 7.
Notes 'cash covered' in pencil against item on CLM.
Types sums in the formula cells of **columns 2 and 3** of row 7.
Amends number in **column 3** remarking "one facility is in credit ".

E reformats table rapidly; in the process of reformatting table he creates and autosums 'net at risk' subtotal and total.
Adds rows 8 and 9 to table typing text and numbers.
Performs the calculation in the formula cell in **column 3**, row 8 using information in table and numbers in **columns 3** of rows 7 and 2.
Uses the autosum function to put total in **column 3** of row 9.
9.10am Saves copy of template for observer.

The end point was this summary:

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[1] (5) and (6) on figure 1.
[2] 'fx' means foreign exchange.

Exhibit 3 – Extract from the Wallaby credit review working paper

Current facility position

Nos	1	2 Facilities* £'000	3 Outstandings* £'000
4	Foreign Curren[c]y loans	15,173	13,797
1	Bonding lines (no cash cover, 50% LC** from X Bank)	3,000	2,409
7	New Bonding facility (50% cash cover)	18,750	6,047
2	Bonding lines (cash covered)	54,000	37,893
3	Trading lines	1,796	-
5	Other	5,790	-
	Net at risk	**98,510**	**60,146**
6	Settlement facilities	10,000	-
	Total	**108,510**	**60,146**
8	Cash collateral		(40,916)
9	**Total exposure**		**19,230**

Footnotes: The first column was added to Eric's original table to aid discussion. The red numbers indicatethe order in which rows were first typed.
*'Outstandings' are loans due to Bank at the date of the review and 'Facilities', maximum loans authorized by the Bank's Credit Committee.
**Letter of Credit

Eric took about twenty minutes to compile the table. He created a similar table for each group subject to review. Generally the information in the tables was derived from three, rather than two, Bank documents – the borrowing group's facility application, the attached review of operations and the CLM for the group.

7.3.3 Why the table was produced

The task in hand was to present and summarize the Wallaby credit position at the date of the review in a form that is easy to assimilate. Eric constructed the table because the CLM produced by Bank neither summarized nor totalled the balances and it was not easy for the audit team to extract meaningful information from the listing (source: Cliff). As Eric composed the table, he grouped loans together and created the loan classifications; in the process he created new meanings, meanings more relevant for the audit.

The table shows the totals (1) of authorized lending limits for facilities and of the loans outstanding before and after deduction of the value of the collateral held by the Bank at the date of the review. The net amount of the loans outstanding (2) is calculated by deducting the total of the cash collateral (3) provided by Wallaby from the loans total. The table also shows that the total of the loans outstanding in any row is less than the authorized limits (4). To ease understanding of the commercial nature of the facilities, those of a similar nature are grouped together (5) and the categories are named and ordered (6). Arguably (6) is not mathematical in nature. The text in the table attaches commercial meanings to every number displayed. Together they summarize, order and give meanings to the balances. Thus the table simplifies and organises complexity. The mathematics used assists; following Luhmann, it acts as a code[1] to assist communication.

7.3.4 Aids to cognition; structuring artefacts and practices

The CLM, which showed all balances due to the Bank and its fellow subsidiaries from the Wallaby group, structured Eric's classification of the balances into categories (lines 7-12, 22-23 and 34-39 of observation notes) and the initial order (lines 12-16 and 18) in which the categories appeared in the table. As Eric inserted the balances from the CLM into the proforma and totalled them, he transformed it into the summary table. The CLM both substantially determined and structured the input to the table. Eric used the equivalent table in the half year working paper to re-assess the order in which the balances appeared in the table (lines 28-30) and the Bank review to summarize the nature of the security for the loans.

Technology comprising Excel and an Excel proforma spreadsheet embedded within a Word proforma document facilitated Eric's creation of the summary of the Wallaby loans:

- The structure of the Excel proforma substantially determined in advance the form but not the content of the table. Furthermore
- Excel provided a standard high level computer language for arithmetic and algebraic expressions.
- It performed all the calculations.
- It also facilitated record keeping and the process whereby the input to the calculations are checked for accuracy and completeness as all numbers could and were recorded individually in the table or in its formula cells.
- The electronic form of the spreadsheet stored a record of all the calculations. And
- The use of the Excel spreadsheet eased the input, correction and manipulation of the data.

Computer technology thus provided some saving of cognitive effort and enhanced both the presentation and record keeping.

I also suggest that Eric's performance was improved because the preparatory work of setting up the proforma and his practised use relieved him of working memory overload when he was constructing a table for a particular case. It enabled him to concentrate on facts that were particular to the Wallaby case, e.g., the quantum and form of the collateral (security) for the loans.

But one must not forget old technology, the auditor's ubiquitous pens and the routines of checking off numbers. As Eric inserted a number in the table or checked it, he

[1] "The combination of information, utterance, and expectation of success in one act of attention presupposes 'coding.' The utterance must duplicate information, that is, on the one hand, leave it outside yet, on the other, use it for utterance and reformulate it appropriately:......What is sociologically important is, above all, that this too brings about a differentiation within the communication process. Events must be distinguished as coded and uncoded. Coded events operate as information in the communication process, uncoded ones as disturbance (noise)." (Luhmann, 1995, p.142)

ticked it off on the CLM using differing colours for each series of ticks. As well as providing a permanent record of work done, this longstanding audit practice saved Eric from having to remember which numbers had been put into the table and which had still to be inserted.

Eric used cognitive artefacts (the CLM and the Excel proforma) and processes (categorizing, recording, ordering and checking the balances) to organise and summarize information. They saved him cognitive effort and they facilitated and ensured the quality of his output, partly because much of the routine was well practised and partly because cognition was distributed across the subsystem both temporally (e.g., use of the proforma and the results of past cognitive effort locked into Excel and Word programmes) and spatially (e.g., the coordination of the CLM with other papers and the summary table). By summarizing the information in the CLM, Eric saved himself and the team time and future effort.

Eric's coordination with the CLM and the Excel proforma, which in due course became the Wallaby summary table, provided the substantive content and structure of the table (1 to 3 in figure 1). Other resources played a supplementary role (4 to 8 in figure 1). Nevertheless Eric brought considerable skills and knowledge to the task. He read and extracted meanings from Bank and audit papers. He extracted and summarized information for insertion into the table and in the process created new meanings. He composed and typed calculations into the formula cells, instructed the computer to perform calculations and total columns of figures. He re-formated the table to customize it to the facts and circumstances of the Wallaby facilities and loans. He checked the table for completeness and accuracy and he assessed the result for reasonableness – see exhibit 4.

7.3.5 The structure of the Excel proforma spreadsheet and the embedded mathematics

The proforma (exhibit 1) was designed by Eric for use in the year end loan loss review. It was based upon some of those used in previous reviews.

The use of the Excel spreadsheet generally determined the form used to express mathematical expressions (7) and the programme performed all the calculations (8).

The structure of the proforma sets out how the numbers are to be presented, resulting in a presentation that is parsimonious without sacrificing detail deemed essential from an audit perspective. Both Tom and Cliff commented on the usefulness of the summaries. The proforma sets a number of mathematical features for the summary tables:

- the loans are to be valued in £s sterling, determining the conversion (9) of non-sterling loans to sterling;
- the loans are to be rounded to the nearest thousand pound (10) – the rounding is coded (11) via an Excel function into the format of the proforma;
- facilities are to be classified and subtotalled (12) by type;
- numerical comparisons (13) are to be made between authorized facilities and the amounts outstanding;
- columnar addition/subtraction is to be used (14); and

- totals of facilities and outstandings net of security before and after inclusion of settlement facilities are to be computed **(15)**.

Through the construction of the proforma Eric defined what he believed the summary tables ought to show, including the mathematical relationships **(9) to (15)**. Standard algorithms of accountancy – columnar addition and subtraction and side by side comparisons – were used.

Many but not all of the meanings of the calculations are made explicit. In particular, neither the proforma nor the summaries make explicit that the purpose of the comparison to be made between the facilities and the outstandings is to demonstrate that amounts outstanding are less than and hence within the authorized limits; the reader is left to draw this conclusion. One key feature was absent from the proforma – a row to show the amount of any specific provision held against the loans; Eric added this when he constructed particular tables. Also when constructing the tables I observed, Eric altered their structure so the value of any security was deducted from the total arising after, rather than that before, addition of settlement facilities. Given the speed with which he changed the format of the tables, it is probable that he had decided that the revised form was more appropriate but had not encoded the changes into his proforma. Otherwise Eric followed the structure provided by the proforma.

7.3.6 Eric's auditing practices summarized

Eric classified the facilities into subgroups, transferred the facility and loan details from the CLM into the summary table, typed details of the cash collateral backing the bonding lines (obtained from the Bank review) in brackets in cells of column **1**, totalled the authorized facilities and the outstanding loans, and finally calculated the quantum of the cash collateral and deducted it from the total of the outstanding loans. There were two significant features to Eric's classification system: bonding lines were classified according to the extent to which they were cash covered and Eric moved the foreign currency loans which were issued on the reconstruction of Wallaby so they led the table.

Two of Eric's auditing practices have significant mathematical practices embedded in them.

As Eric inserted the numbers into the table, he ticked **(16)** them off on the CLM. And when he reviewed his first draft of the working paper, he systematically checked each number in the CLM to the electronic copy of the table again ticking off **(17)** the numbers on the CLM. Eric thus checked the input to his calculation for both accuracy and completeness – an important step to ensure that his calculations are correct.

After Eric had completed rows **1-6**, he turned to the summary of the position in the working paper for the half year and carefully checked the entries in that summary against those on screen in the current working paper. During this process he also referred to the CLM. The numbers in the two tables were slightly different as were the descriptions and the classifications. After Eric completed this task he moved the foreign currency loan line to the top of the table, and finished inserting information in the table about bonding lines and the cash collateral. Eric expected there to be strong similarities between the two positions as both summaries referred to periods in 2003 subsequent to a major reconstruction. I believe that here Eric was comparing numbers

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7.3.7 Computational practices

All computations, other than roundings and comparisons, were executed electronically by the Excel programme, after Eric inserted each tranche of data. The table only displayed £000s as formatting hid hundreds, tens and units.

Classifications and numbers (primary data) were taken from the annotated CLM and inserted in table. The data was entered in cells of columns **1, 2 and 3** of each row in turn – Eric rounded (19) the numbers to the nearest thousand by inserting a decimal point after thousands place value position.

If only one number was due to be entered into a formula cell in the table then, with one exception, Eric typed the number straight into the table rather than into the Excel formula cell. Otherwise he typed (20) a calculation into the Excel formula cell. For loans or facilities in foreign currency, the quantum of the loan or facility was typed into the formula cell and converted to sterling by dividing (21) by the foreign exchange rate (fx rate), which was noted on the CLM. Subtotals and totals were calculated using the Excel autosum function (22).

The cash collateral figure was calculated after reading the text in brackets relating to the bonding lines (1, 7 & 2). The calculation in the formula cell, which was composed by Eric, reads as follows: = (C7+(C6*0.5)) (23). This is the cash collateral = cash cover for bonding lines reported in rows 1, 7 & 2 = [0% of 2,409 +100%] of 37,893 + 50% of 6,046. Eric derived the calculation from reading the information he had set up in the table and used spreadsheet algebra to set up the calculation, the latter being more efficient, less subject to error and in the event of subsequent alterations to figures self correcting. It is the most complicated calculation in the table.

In this paragraph, x,y,z replace the numbers in Eric's calculations. All numbers were rounded as described above. Eric composed and typed the following calculations into the Excel formula cells:

= (x+y) / fx rate	twice (21)
= x+y	twice (20)
= x / fx rate	once (21)
= x / fx rate + (y+z)	once (21 & 20) (here 'x/fx rate +' was inserted after the rest of the calculation had been set up)
= (C7 + (C6*0.5))	(23) the calculation of the cash collateral
and	
the autosum function key	was used five times (22), once performing subtraction

As Eric constructed the table, he made one error. Immediately after he had inserted the information in row 7 he noticed one balance on the CLM was a negative number. He deleted (24) that number from the formula cell remarking "one facility [is] in credit", meaning the balance was not an amount due to the Bank but an amount due to Wallaby.

Although all calculations (other than roundings) were performed electronically, Eric controlled the whole process; he typed the instructions for the calculations in the formula cells and instructed the computer to carry out the computations. He also made sure that the input was both complete and in the correct format.

7.3.8 Using the summary

The team used the summary tables (both the half year and year end ones) to make sense of the Bank's exposure and to extract meanings.

As Eric constructed the table he looked at the equivalent table in the half year review to help him both to make sense of the current period's exposure and with the construction of the table. Eric and Cliff reviewed the summary.

Exhibit 4 – Modified extracts from my observation notes; using the table

After Eric finishes compiling table, he looks at table and says to [observer] "Total 19.2m. Therefore I am not too excited. I will comment on the gross position, that is [the position] before [deducting] cash collateral."

When Cliff looks at table for first time, he says " Um 20m…….well that's quite small, smaller [than] I thought it was." Cliff and Eric discuss briefly how the Bank's exposure has been reduced.

Cliff then focuses on the rows setting out the information relating to the bonding lines and says

"And has some one looked at the…. collateral?………"

As he and Eric look at the table, Eric explains what is covered by collateral:

C……..And make sure that the cash collateral covers all the lines.
(Cliff and Eric are looking at detail in table not totals)
E Yup well only covers….only covers the bonding lines.
(Short pause – both continue to look at table)
E And yeah **just basically covers the bonding lines**.
C In **total**?
E Well in um **Covers 50%**. **They have got a 50% LC** [letter of credit] yes **for those two 50%** of that (both looking at table and Eric points to rows 1 & 7) and **full** [meaning 100% cover] on that (Eric pointing to row 2).
E And **then nothing** on that (Eric pointing to row 4)
C The foreign currency?
E The foreign currency loans.
C Oh OK

And as Cliff comes to the end of his review of the table, he says "And that's the cash collateral that we hold? Nothing complicated?" Eric answers, "Nothing complicated."

When Cliff and Eric looked at the summary, they focused first on the numbers (25) as they made sense of the credit position. Significant numbers in the table served to focus the sense making and discussion. They both referred (26) to the total net amount outstanding, rounding (27 & 28) to one decimal place to the nearest million and the nearest tens of million respectively. Rounding to relatively few significant figures in conversation was a frequently used practice.

Eric had spent time reading about and understanding the position relating to collateral in the CLM and the Bank's review of operations and had summarized the position in the table. Cliff (see above) paid particular attention to it as he reviewed Eric's working paper. Eric read and re-understood the position and quantified **(29)** for Cliff the cover for each of the main lines in the table using percentages. Cliff reads, listens and signifies he understands by saying "Oh OK" **(30)**. (Tom also focused on the collateral at a later stage.) Eric and Cliff focused on the numbers – see bold type in exhibit 4.

Eric and Cliff read the tables, extracted and clarified meanings.

7.4 Interim conclusion; Embedded mathematical practices constitute *workplace mathematics*

The purpose of the table is to summarize and total the Bank's loans to Wallaby at the date of the review. The table classifies the loans into categories which are meaningful from an audit perspective and shows all outstandings are within authorized lending limits. Mathematical relationships **(1 to 5)** are used to present a picture and to assist in ordering and simplifying the complexity of the situation.

Many of these and other mathematical relationships, i.e., *workplace mathematics,* were locked into the proforma **(9 to 15)**. From a *workplace mathematics* viewpoint other important technological gains were derived from using the Excel programme;

- a high level computer language for writing arithmetic and algebraic expressions;
- automation of calculation; and
- transparency of data input to the electronic form of the summary tables.

The spreadsheet proforma devised by Eric was substantially fit for purpose even though it did not adequately deal with the quantification of specific bad debt provisions or, to a lesser extent, the value of any security provided.

As Eric worked he typed **(19 to 24)** instructions for calculations into the formula cells – seven in all – and instructed the programme to perform all calculations. Eric showed proficiency in creating 'mathematical sentences' using spreadsheet arithmetic and algebra. He made some minor amendments in the course of constructing them. Eric used other mathematical practices. He rounded numbers and checked the accuracy and completeness of calculations. In particular, he made two sets of comparisons – one to test whether the outstanding borrowings were less than the authorized lending limits and the other to assess the similarities (in structural terms) between the half year and the year end position **(13 & 18)**. He also analysed, classified, grouped and re-described the balances which were inserted into the table. The latter practice combined both mathematical and commercial objectives in that the purpose was both to quantify and attach suitable commercial meanings to the subtotals, but otherwise all was *workplace mathematics*. He also checked the input to calculations for completeness and accuracy **(16 to 17)**.

Once the table was completed Eric and Cliff interpreted the results using *workplace mathematics*. They focused on the figures and their meanings (26, 29 & 30), starting with the totals and then on the nature of the collateral. In conversation the totals were rounded up to millions and tens of millions (27 & 28) – the table they were reading from was rounded to the nearest thousand.

Throughout we have a substantial number of examples of *workplace mathematics*. Mathematics was complexly embedded and interwoven into the performance of the task. It is also noteworthy that, throughout, commercial meanings were kept attached to the numbers and the operations as Eric and Cliff manipulated the data; I shall return to this in the next chapter.

I now consider the construction of another table – one that was driven solely by the particular facts and circumstances of the case under consideration and then compare and contrast the two examples.

7.5 Possum – re-presenting the credit position on an entity basis

7.5.1 Background

The construction of the table in exhibit 6 (refer to as 'the Possum table') was designed to show the split of the Bank's loans to the Possum group across legal entities. Its construction was driven by the particular facts and circumstances of the Possum group. It is of interest because it is a much less polished presentation that the Wallaby summary table and Eric used slightly different resources to construct the table, namely a summary of the debt position from the Bank's review of operations, a Word table and a calculator. But first, I describe the background and the facts and circumstances that led to the construction of the table.

Possum was one of the five loan loss reviews observed. Possum Inc (Inc), a US based utilities business, was in financial difficulties in 2003. In 2002 it had acquired PEPH, a UK holding company, which owned Platypus Electricity (PE), which in turn owned Possum Networks (PN). PE sells electricity to customers in the Midlands while PN supplies the electricity to PE. PN also generates a small amount of electricity in the UK, Turkey and Pakistan. All the companies were heavily in debt and there were doubts about the ability of all the companies, but particularly Inc and PEPH, to meet their debt obligations. The credit ratings of PE and PN (FG4 using the Bank's notation) were higher than those of Inc and PEPH (FG5) reflecting the fact that the risks attaching to loans and other credit exposures of PE and PN were less than those attaching to Inc and PEPH. Neither PE nor PN had provided guarantees to other group companies; they were prohibited from doing so by Ofgen, the UK electricity regulator.

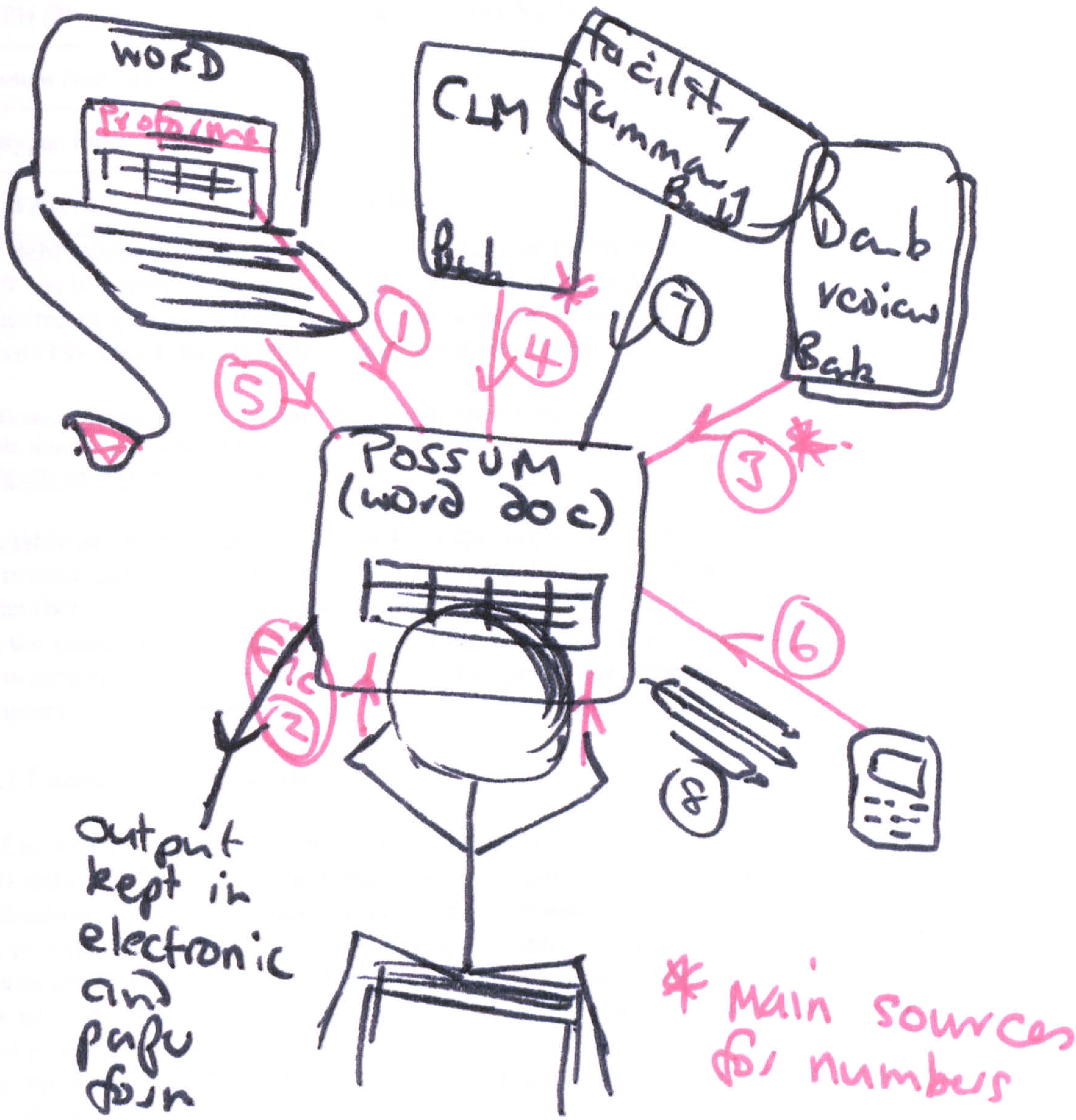
The Bank had lent money and provided facilities to the UK group. Using the Possum CLM, the Bank review and the Excel proforma, Eric summarized the overall position as follows:

Exhibit 5 – Extract from Eric's working paper; the Bank's exposure to the Possum group

Current facility position	Facilities £'000	Outstandings £'000 @13/11/03
On balance sheet	60,000	35,269
Treasury products	31,494	504
Off Balance sheet	4,900	2,716
Other	56	-
Net at risk	**96,450**	**38,489**
Settlement facilities	120,200	-
Total	**216,650**	**38,489**

The Possum group was not reviewed at the half year. Therefore Eric had little prior knowledge of its position before starting his review. After constructing the Possum summary table (exhibit 5), Eric referred to the facility application and read the Bank review of operations fairly carefully. It comprised six pages of commentary and analysis – it was not easy reading. As Eric read the first three pages of the review, he highlighted some key points using an green highlighter and wrote most of the sections in his working paper headed 'Current key issues affecting status of credit' and 'Reason for original non performing status'. This took about 40 minutes. He then stopped writing and spent the next 15 minutes reading the rest of the review, pausing to reflect from time to time. During this process he decided to construct the table in exhibit 6 but before he did so he wrote the following in his working paper: "*The majority of the exposure [is] to [PN]*. Only the PEPH facilities are graded FG5, the other facilities are graded FG4 the reason being that PEPH is reliant on the dividends from it's (*sic*) subs PE and PN." (Italics are mine.)

Schematic diagram of Figure 2
Eric working at his desk
constructing table showing
split of facilities | loans to POSSUM



all necessary to task

supplementary information | tool

→ input ← output

Exhibit 6 – Extract from Eric's working paper; debt of the UK Possum subgroup

Current facility position

(Exhibit 5)

The debt can be broken down as follows (1), Bank debt excludes settlement balances (9):

	(2) Total debt	(2) Bank Debt*, £m		(2) Outstandings, £m	
PEPH (2)	(3) 661.3	(11)(USD 5m)	(6) 3	(12) (USD 12k)	(13)-
Possum Networks (2)	(4) 520.7	(7 & 8) 75		(14) 35	
Platypus Electricity (2)	-	(10) 18.5		(15) 3	
TOTAL (21)	(5) 1,182	(17) 96.5**		(16) 38**	

No[1] debt exists at an Platypus Electricity level, or in any of it's subs (18). A bond of £149.5m is issued in it's name (Bank participation = £nil), however this is subject to an upstream guarantee from Possum Networks and has been included in it's total debt above (19). check for classification of PE debt in Bank (20)

Footnotes: The numbers in red show the order in which table was constructed.
* This should read authorized facilities not debt, meaning obvious to informed reader.
**Figures conformed to 'Net at risk' totals in exhibit 5.

The table in exhibit 5 shows the Bank's exposure to the Possum group at the time of the review and the table exhibit 6 shows the total long term debt of UK group at 31st December 2002, the Bank's authorized facilities (excluding the settlement facilities) and the outstanding balances to the group at the date of Eric's review. Eric's purpose in constructing the latter table is to show the split of the outstanding balances by company – see discussion below.

7.5.2 Constructing the table

As Eric started constructing the table, he had all the resources that he needed at his workstation – his PC with the Possum working paper on screen, the facility application with the Bank review attached, the Possum CLM, his calculator and pens and pencils. See figure 2. He typed below the table in exhibit 5 "The debt can be broken down as follows" and then he set up a 4*5 Word table. The structure of the new table is derived mainly from a more detailed table in the Bank review. As are most of the numbers, the main exception being that Eric used the outstanding balances from the Possum CLM so that they reconciled with those used in exhibit 5. The table took about 20 minutes to construct.

My description broadly follows the order in which the table was constructed. The numbers in the table were derived from the table in the Bank review, unless otherwise stated. They have been rounded (A) to millions to one decimal place following the table. The figures (3), (4) and (5) are subtotals and were copied from the review. Before Eric inserted (5), he checked (B) the facilities shown in the review against

[1] Here 'no' means 'no material' or 'no significant'.

those he had marked up in the facilities application. Eric spent some time trying to understand why a £149.5million bond issued by PE had been included in PN debt. He commented: "All very odd. These fall under Possum Networks.....[I] propose checking [the] other..[numbers] first". The note at the foot of the table (18 & 19) was typed after Eric had completed its construction. Eric filled in the column headed 'Bank debt' using the numbers from the review except that the $ amount came from the facility application (11 & 12). Using the Bank review table, Eric totalled all the authorized facilities for PN on his calculator and entered £95m (7) **(C)**. He referred to the Possum summary table and decided **(D)** the two tables should show the same net at risk totals (see exhibit 5) and so amended the PN figure to £75m (8) **(E),** excluding the £20m settlement facilities of PN. He added the explanatory caveat to the table heading (9). Eric totalled the figures in the review for PE's facilities (excluding those he had marked as being settlement facilities) on his calculator and entered them in the table (10) **(F)**. Eric then filled the figures in the outstandings column using the Possum CLM as the source. The figure for PN (14) **(G)** is the sum of two figures and was estimated mentally, while the figure for PE (15) **(H)** is a sum of two figures and was totalled on the calculator and rounded to the nearest million. (Eric ignored **(I)** one figure because it was under £10,000 and therefore was not material.) Eric entered the totals in table; (16) **(J)** was calculated mentally while (17) **(K)** was computed on the calculator. Eric checked **(L)** that the totals reconciled to those in the Possum summary table.

Eric decided to construct the table to show that the Bank's only significant exposure was to PN and hence justify his decision to review only PN's accounts. Immediately after Eric finished the table, he went to the box in the working paper titled 'Financials' and wrote "Due to the low level of exposure on PEPH and PE we will only review the financial statements of PN." On a stand alone basis, the table's purpose is not self evident, particularly as it contains superfluous information (e.g., details of group debt) and a possibly misleading description ('Bank debt' instead of 'Bank facilities'), making it neither quite 'fit for purpose' nor parsimonious in presentation. However Eric saved considerable cognitive effort by adapting and abridging a table from the Bank's review of operations and its purpose when read in conjunction with the rest of the text is clear. The key results shown in the table order and simplify the indebtedness to Bank and justify the decision to look only at the affairs of PN, thus considerably simplifying the main task in hand. In constructing the table and using the results it showed, Eric engaged in *workplace mathematics.*

7.6 Comparing and contrasting practices

7.6.1 Comparison of the Wallaby and Possum tables

a) Why the tables were produced

1) The Wallaby table summarized the Bank's credit exposure to Wallaby at the date of the review. The purpose of the Possum table was quite different; it was designed to show that most of the Bank's exposure was to PN and therefore that the audit team needed only consider its financial position in depth.
2) The Wallaby table was self standing while the Possum one was not. The analysis of the Bank's outstanding loans on a company by company basis justified Eric's decision to analyse PN's results only. The table contained superfluous information

with the result that its intended purpose was not clear without reading the text of Eric's paper.

b) The tables and structuring resources

3) The main structuring resources were different though comparable. The Wallaby table is an Excel spreadsheet with the information in the table being derived mostly from the Wallaby CLM, while the Possum table is a Word document with both its structure and content being derived mostly from a table in the Bank's review of Possum's operations. The available resources had a substantial impact on the form, construction and content of the tables.
4) To construct the Wallaby table, Eric used the Excel proforma he had designed for use in all the loan loss reviews. This determined the presentation, the form (typed into the Excel formula cells) and method of calculation – for details see earlier discussion. The position with Possum was different. Eric decided to adapt and abridge the table in the Bank's review of operations; this table not the Word table structured his presentation. It determined that the balances were reported in millions to one decimal place and that the total indebtedness of the UK Possum group was included in addition to the Bank's own position.
5) For the Wallaby table, all calculations were performed by the Excel programme while for the Possum table computations were taken from originating table, performed on a calculator or, in the case of two easy ones, performed mentally.
6) However similar accounting conventions and practices were used to construct the tables. Both used a row/column format. Specific meanings were attached to information inserted in the rows and columns of the table. Information in the rows allowed comparisons to be made between items while generally the totals and subtotals in the columns were of particular significance. All figures to be subtotalled and totalled in the tables were aligned vertically; they were set up to facilitate the use of columnar algorithms for addition/subtraction or the Excel autosum function key. Also the numbers in the tables were rounded to different degrees of accuracy but both were 'fit for purpose'.
7) The Wallaby table was essentially stand alone. Eric did not regard the Possum table as such. Eric conformed it to the Possum summary table. To do this, Eric decided not to include the settlement facilities in the totals and to take the balances for the outstanding loans from the Possum CLM, rather than from the Bank review. He resolved a possible presentational ambiguity as he constructed the table.

c) Completeness and accuracy

8) The electronic form of the Wallaby table captured all the numbers taken from source documents. This eased checking the input to the table for both completeness and accuracy. The Possum table did not, being in Word. By conforming columns 3 and 4 of the Possum table to the Possum summary table, Eric effectively checked the key parts of the table for both completeness and accuracy.

d) Mathematical practices

9) Both arithmetic and algebraic expressions were composed and stored in the Excel spreadsheet while the Word table only recorded the results of calculations made elsewhere.
10) The mathematical practices used by Eric are described in detail in sections 7.3 and 7.5 above.
11) It is worth mentioning again the use made of comparisons. They were used in four different ways; to show that the loans outstanding were less than the corresponding authorized limits, to ensure the completeness of input to the Wallaby table, to exclude the PN settlement facilities from the PN balances so the Possum table would conform to the summary and to reconcile the Possum table and the Possum summary table.
12) Eric probably used the Word table as he did not expect the creation of the Possum table to involve much calculation; this proved to be the case. This is to be contrasted with the PN cash flow which is on page 3 of the Possum working paper (exhibit 9, section 8.4) and for which Eric used an Excel spreadsheet; Eric would have expected the calculation burden to be quite heavy for a cash flow.

The Wallaby table provides a complete, accurate and meaningful summary of the Bank's exposure to it. By contrast the intended purpose of the Possum table cannot be deduced from the table alone and its presentation is not parsimonious. Superficially the construction of the Possum table appears to be more straight forward than the Wallaby table. It is arguable that the difference in quality derives partially from the fact that Eric had created a proforma for the loan summaries and that Eric's expertise in creating the summary tables, which was derived from practice, considerably reduced the cognitive effort involved in the construction of the Wallaby table.

7.6.2 Comparable findings from chapter 5

I now consider the participants' calculation practices described in chapter 5 briefly. Holly, Gary and Sasha all read financial information and extracted meanings from it. They also used calculators or the Excel programme to perform all but the simplest calculations. Holly and Gary checked numbers in tables/accounts for completeness and accuracy of texts created by others. Gary created reconciliations of numbers where one number could not be checked directly against another, including a relatively complicated proof of the managing director's remuneration. Sasha set up an Excel spreadsheet to compare the value of programmes transmitted per four different sets of company records. In setting up this spreadsheet she composed 10 calculations using Excel function keys and used the autosum function key to sum all totals. Sasha rounded numbers to the nearest thousand in the accompanying text to her table but did not do so in the table itself. Gary's use[1] of calculation as he sought to understand the make up of managing director's remuneration was qualitatively different from Eric's usage. However, in the course of this exercise he typed a number of relatively (cf. Eric's usage) complicated calculations into his calculator. Thus the participants calculation practices described in chapter 5 are not dissimilar to those used by Eric as he constructed the two tables.

[1] See section 10.3 for discussion of exploratory calculations.

7.7 Concluding remarks; Eric thinking and doing

As Hutchins hypothesised, Eric's work is influenced by his working environment and much of his cognitive effort is evidenced by his actions and the texts he created and only occasionally is there a need to speculate about his mental processes.

We see Eric presenting financial information in tables which are end products. The two tables, however, serve different purposes. The decision to create the summary table for Wallaby was driven by past practice, though the design based upon previous year's tables was Eric's specific design. Eric decided to create the Possum table as a result of the particular facts and circumstances of the Possum case. He simplified his task by abridging and adapting a table from the Bank's review. The creation of the tables was facilitated by the available resources, past practice and routines and Eric's own knowledge and skills.

Mathematical codes are used to organise and simplify specific facts and circumstances while the relationships between numbers provide connections from which commercial meanings can be derived. Mathematical practices encapsulated in the tables thus serve as a code to organise/reduce complexity and are used to aid the presentation of meaning and in the case of Possum simplify the main task in hand. We also see how Eric and Cliff used the results presented.

Eric designed and used the Excel proforma and created the Word table. The particular tables, as explained above, incorporated mathematical relationships and were substantially structured by the particular resources Eric used. Other mathematical practices, i.e. *workplace mathematics,* were used as the tables were compiled:

- Eric himself constructed almost all of the calculations to be performed – the main exception being the calculations copied from the Bank's papers for the Possum table.
- Virtually all calculations were, on Eric's instruction, performed electronically.
- Eric often checked input to tables and other calculations for completeness and accuracy.
- He used all four arithmetic operations and composed an algebraic expression as he worked, mostly typing them into the Excel programme or a calculator.
- All numbers were rounded to an appropriate degree of accuracy.
- Eric also made a series of comparisons as he worked.

There is a detailed summary for the Wallaby table in the interim conclusion (section 7.4).

Eric used a wide range skills and knowledge of mathematical practices as he created and used the tables, including reading and making sense of a range of texts. Eric made many choices as he worked, shaping both his understanding and his output. A subtext that emerges from this chapter is the critical importance of Eric's own knowledge and to skills to the work observed.

The analysis shows that Eric used *workplace mathematics* extensively. It also shows that *workplace mathematics* is complexly interwoven into work practices, the texts used and produced, and the artefacts (including those of an intellectual nature) used to accomplish the work in hand. The analysis shows in particular:

- the purposes served by the tables;
- how Eric's work was structured by the artefacts which were in and were part of the setting;
- the mathematical structures that are locked into the tables;
- some of the mathematical skills in auditing practices observed; and
- the computations performed.

These two relatively straight forward examples tell us much about the *workplace mathematics* observed and its relationship with the task in hand and the structuring resources, and Eric's skilled usage.

They also highlight two common products of *workplace mathematics* within the financial community – namely the use of mathematics to assist in the presentation of information and its use to order and simplify information and tasks in hand. Finally we also see Eric and Cliff reading the Wallaby table, extracting meanings from the table, clarifying and interpreting them.

8 Financial models: *workplace mathematics* in action

8.1 Introduction

In this chapter, I describe how my participants used accounting models which are also mathematical models. The emphasis here from a *workplace mathematics* viewpoint is on the construction and use of mathematical models, rather than on calculation practices. In section 2, I look at the discursive use of a debt valuation model. This shows, in particular, how *workplace mathematics* is interwoven in discussions between team members. In sections 3 and 4, I describe and analyse the construction and use of cash flow models by Tom and Eric respectively. Tom's analysis of the Kookaburra cash flow is used to unpick an expert extempore performance. By way of contrast, Eric's construction of the Possum cash flow is used to show how a model was constructed by a newly qualified accountant, a relative novice. I also speculate about the possible mental models he used. In section 5, I discuss aspects of the construction and use of a goodwill model to illustrate the minutiae of model building. In section 6, I draw together some key themes that emerge from the analysis.

8.2 The bond valuation model – assessing the Kookaburra bad debt provision

I follow the team's use of the bond valuation model (a mathematical model) as they progressed their review of the Bank's loans to Kookaburra. Through discussion focused primarily around the model, the team of Tom, Cliff and Eric moved from Eric's preliminary position that the provision did "not appear to be materially misstated" to the position that it was considered to be understated. I show how in meetings Eric, Cliff and Tom each first understood the model, used it and then finally in a drafting meeting together arrived at their judgement that the Bank was underprovided. The discussion illustrates how cognition was distributed between team members and was situated in the facts and circumstances of the case. I conclude the section with a discussion of how the team jointly formed its audit judgement, an activity which incorporated *workplace mathematics,* and the consequences of that decision. I start by describing the background.

8.2.1 Commercial history of loan – pre and post review

The Bank lent money to Kookaburra. In the course of debt reconstructions it acquired equity (shares in the company/Equity notes) and bonds (referred to here as senior and junior[1] debt or debt). The Bank had a provision against its loans to Kookaburra as it considered that part of the debt was not recoverable. During the half year audit, the audit team assessed the adequacy of the Bank's provision to be reasonable, as the value placed upon the debt in the balance sheet was broadly in line with the value calculated using secondary market prices and the market value of the equity owned was greater than its book value. ((Source: manager's conclusion in half year loan loss review.) Subsequent to the half year, the Bank had sold the equity and the prices of the junior debt had fallen in the secondary market.

[1] 'Senior/junior' refers to the priority of a creditor in the event of a winding up.

Following discussions between the audit team and the Bank staff after the loan loss review, the Bank valued the debt in the year end balance sheet using the secondary market prices (i.e., marked to market), thus increasing the bad debt provision. The issue of a potential under provision emerged from Eric's work because his review was carried out before the Bank had started its year end work. Two other issues, not dealt with here, were raised in team discussions: the US accounting treatment (often referred to as US GAAP); and the operating cash flows of Kookaburra (see section 8.3). Both influenced the development of the team's views.

8.2.2 Reviewing the bad debt provision – what the team did

Exhibit 1 – Timetable of events

October/early November 03 Eric drafts credit update paper on Kookaburra.*

13/11 Meeting of team with head of the Bank's loan monitoring unit (LMU) to discuss loan portfolio and large loans giving concern. Kookaburra discussed.

17/11 Eric and Cliff have meeting prior to Cliff's review of Eric's work on some of the loans in the loan loss review. Kookaburra discussed at length.

Cliff reviews the Kookaburra file.*

19/11 – morning Cliff has short meeting with Eric to discuss his review points. Kookaburra discussed briefly. In particular Cliff focuses on differences at the end of the second and third quarters of the financial year.

19/11 – afternoon Tom has two hour meeting with Cliff and Eric. In this first partner review meeting relating to the loan portfolio, Tom obtains an overview of the portfolio, and discusses the first batch of loan reviews. Kookaburra discussed.

27/11 Tom reviews the Kookaburra file and queries valuation of the loans.

2/12 Meeting to agree contents of Audit Committee Paper. The paper is a summary for the Bank's Audit Committee of the key issues that have arisen in the course of the audit. Update on Kookaburra to be included.

9/12 Team meeting to discuss first draft of the Audit Committee Paper. Paragraph on Kookaburra is amended and the audit team arrive at the conclusion that the bad debt provision is understated.

Provision and basis of calculating bad debt provision discussed with Bank staff.*

22/12 Cliff's second review of Eric's files. Does not look at Kookaburra file because the Bank is reconsidering bad debt provision.

January 04 Bank decides to make further provision. The Bank marks loans to market (value at 31st December 2003) and the outcome is reported to the Audit Committee in the Audit Committee Paper.*

*Not observed

8.2.3 Background information from audit papers

Eric's credit update paper is in his standard format and is short. It does not stand alone. In the 'Justification of provision' section, Eric directs the reader to the valuation of the debt in the Bank's schedule headed 'Kookaburra exposure as at 1st October 2003', an annotated copy of which is in the Kookaburra working papers' file. The only material change to the Bank's exposure to Kookaburra between the half year review and Eric's review was the sale of the Equity notes. This is not mentioned in Eric's paper but the details are included in a footnote to the Bank's valuation of the debt.

Exhibit 2 – Extract from credit update paper dated 7th November

Conclusion of prior review	Cash flows not sufficient to repay debt, value of debt based on M[arket] V[alue.]

Exhibit 3 – Extract from credit update paper dated 7th November

Current facility position		
	Facilities £'000 @23/10/03	**Outstandings £'000 @23/10/03**
Senior Debt	xxxx	xxxx
Junior Debt (Tier 1)	2,218	2,218
Junior Debt (Tier 2)	28,213	28,213
Junior Debt (Tier 3)	104,274	104,274
Other debt	xxxx	xxxx
Net at risk	**xxxx**	**xxxx**
Settlement facilities	xxxx	-
	xxxx	**xxxx**
Provisions		(48,500)
Net exposure after provisions		**xxxx**

At the meeting on 13th November with Matt, the head of the LMU, the Bank's loans to Kookaburra were discussed as they had been identified as giving concern. Matt provided the Bank's perspective on the loan:

- in his view, the management of Kookaburra had to face up to the need to restructure;
- the business (as it was then operating) was not viable; and
- in response to a question from Tom as to when the Bank would write down the loan
    - he believed that the Bank would not sell; and
    - he would not like to set a precedent by marking the loans to market.

He also reported that the Bank had sold its shares. (Source: my observation notes.) Matt's views informed the team's discussions as they reviewed the bad debt provision; Eric and Tom mentioned them on separate occasions.

Exhibit 4 – Extract from the Bank's valuation

Mark to Market of Kookaburra debt

Kookaburra [valuation] as at 1st October 2003 (excluding ....)

Tier	GBPm	Secondary Market Price	Market value GBPm	Provisions	Provisions Recoverable
I	2.22	0.86	1.91		
II	28.21	0.69	19.46		
III	104.27	0.54	56.31	48.47	
	134.70		**77.68**	**48.47**	**-8.55**
Col 1	2	3	4	5	6

86.23 carrying value

Footnotes: Part of the Bank's valuation retyped.
Eric's handwritten annotations typed in red, mine in purple.

8.2.4 Description and analysis of the model

Column 2 shows the nominal values and the total of the nominal values of the junior debts held by the Bank (i.e., the amount repayable); **column 3** the market prices for each type of junior debt; **column 4** the market values of each type of junior debt (being nominal value*price in each case) and the total market value; and **column 5** the bad debt provision carried in the Bank's accounts. **Column 6** shows the difference between the carrying value of the debt (being the total nominal value less the bad debt provision) and its market value. Thus the valuation is a mathematical model. If the number in **column 6** were positive, the Bank might consider reducing the provision – hence the column heading 'Provisions Recoverable'. The negative number indicates that the carrying value of the debt is higher than the market value and is suggestive of a lack of conservatism in the bad debt provisioning.

Workplace mathematics As the Bank's presentation of the results of the model was not entirely clear, Eric calculated the carrying value of the debt, using lines to indicate the numbers used in that calculation, and juxtaposing the result beside the market value of the debt to aid comparisons. The model, as adapted by Eric, highlights two key mathematical relationships – the carrying value of the junior debts (**86.23**) and its relationship with their market value (77.68), namely a difference of –8.55.

8.2.5 Using the bond evaluation model

The audit team's task was to assess whether or not the Bank's bad debt provision for Kookaburra was reasonable, not to propose what the provision should be; that is the nature of auditing. The team therefore gradually gathered information to understand what the Bank had done and why, and then it assessed whether the Bank's position is prudent, adequate or inadequate. The debt valuation model was the pivot for the team's work.

a) Eric uses the model – on 17/11

The purpose of the discussion prior to Cliff's preliminary review of Eric's files was to give Cliff an understanding of the loans and the main issues relating to them. Cliff was new to the Bank audit and had little Bank specific background to the Bank's loan portfolio. However he knew quite a lot about some of the companies to which the Bank lent, having carried out credit reviews of other institutions' loan portfolios. Cliff started the meeting by turning to the Kookaburra working papers file and saying to Eric "Right. Do you want to start with Kookaburra then?"

Eric confirmed that Cliff knew the general story of Kookaburra. He told Cliff that the Bank had "a £50 million provision" *(rounded to the nearest 10 million)* against the Kookaburra provisions. He then immediately shifted his focus to the valuation of the debts telling Cliff that

- the provision was based roughly on the market value of the debts;
- the Bank valued the debts monthly, using distressed debt (secondary market) prices; and
- if the value of the debts was not roughly in line with the provision, the Bank adjusted the provision, but if it was, they would not.

(Eric's explanation paraphrased.)
Eric gave no indication of what 'roughly in line' meant. Cliff then quickly elicited from Eric that the senior and junior debt comprised the main exposure of the Bank to Kookaburra and that the Bank intended to continue holding the debt. Cliff looked at the table of indebtedness (exhibit 3) throughout this exchange. The picture shown in the table confirms much of Eric's comments but does not show the relationship between the provision and the market values of the debt.

Workplace mathematics (in italics).Before moving onto Cliff's consideration of the Bank's valuation, it is worth reflecting on Eric's understanding of the model, as communicated, and his use of *workplace mathematics*. His comments were more nuanced than those in his draft paper, in which he had written 'The provision is based on the M[arket] V[alue] of the debt. ....'. First he signalled that the valuation model is the key factor for the Bank in assessing the reasonableness of its bad debt provision. Second he described to Cliff how the valuation was carried out, *indicating that he understood how the model was constructed* (also demonstrated in exhibit 4). Third he described how he believed the Bank used the valuation model; the Bank made an adjustment (i.e., changed bad debt provision) if the market values and the carrying values of the debts were **not** roughly in line, and none, if they were. *This presents the Bank's usage of the valuation in the form of a logical argument.* The debt valuation model could be used in one of two ways: it could be used so that the carrying value of the debt is adjusted to the market value, i.e., marked to market; or it could be used together with other commercial/accounting considerations as a guide as to whether or not an adjustment should be made to the carrying values. Eric indicated that the latter was then the Bank's practice.

In a short exchange, Eric signalled the pivotal role the valuation played in evaluating the reasonableness or otherwise of the bad debt provision. He also showed that he understood how the model was constructed and how the Bank used it.

b) Cliff looks at and understands what the model is showing – on 17/11

As can be seen from this extract of the transcript of Cliff's and Eric's meeting, Cliff looked at and understood how the Bank's debt valuation worked.

Exhibit 5 – Extract of transcript of meeting between Cliff and Eric, including observation notes (in italics)

C And what's the traded price at the moment? Relative to?
E Um. *(Eric turns over several pages in file)* This one.
(Takes valuation schedule out of file).
E Different debt different prices.
C (*Whispers*) OK.
E There's the debt prices there *(pointing to column 3 in schedule).*
Pause as Cliff looks at schedule.
C Which way round is that? Is it an under or over provision? If you took the market value as the
E Um So that's *(pointing to total of column 4)* the market value of 77. They have got a provision of that *(pointing to column 5)* so there.
C So that *(pointing to total of column 2)* less that *(pointing to total of column 5).*
E Yup.
Pause – quite long.
C So then their carrying in the balance sheet is £9m higher than the market value is that what you they are saying?
E Um *(slightly non-committal).*
Pause
C 135 less 50...
E Yeah...
C is 85. That's the 77, 78 *(referring to market value column).*
E Umm *(short pause)* Yeah, that's right.
C OK *Pause – quite long* OK Thank you *(different tone – closing tone).*
E *(Volunteers)*They're are not marking it to market in the balance sheet
C No no. Yup They are looking to that as an indication.
E Yup.

Several things occur in this episode.

Cliff sought and gained understanding of the valuation (lines 7 to 15) and confirmed his understanding of how he believed the Bank used the valuation (lines 23 to 25). In the process, he also ensured that he and Eric had a common understanding of the position (lines 15 to 21). Eric assisted Cliff as he struggled to understand (lines 6 to 11) with Cliff then seeking confirmation of his understanding from Eric (lines 14 to 21); achieving a common understanding was a collaborative effort.

Workplace mathematics Initially Eric guided Cliff through the valuation by showing him how the market values of the debts were calculated. Cliff then took over and finished the tour by identifying that the carrying value was the difference between the totals of **column 2** and **5** giving rise to the carrying value being £9 million greater than the market value (probably 86 – 77 = 9; evidence in lines 15 and 16). Eric's non committal response led Cliff to justify his calculation of £9m; he showed Eric how he got there "135 less 50........is 85 [compared to] 77...78" After a short pause Eric confirmed he agreed with Cliff's understanding. Cliff in justifying his conclusion of the £9 million rounded the figures to the nearest 5 million, performed a mental calculation and suggested a comparison with 77 and 78 million (rounding the numbers again). This did not replicate how Cliff calculated the £9 million in the first place but revealed a method of calculation which would give the desired result. In the process, Cliff effectively performed a logic check rather than a recalculation. Eric had to perform his own calculation/check. (Evidence is insufficient to work out what he did.) Cliff's calculation practices are similar to those described in chapter 7, except that he actually performed the calculations.

Cliff concentrated on the mathematical nature of both the structure of the model and its results as he pursued understanding of the position. Thus *workplace mathematics* was central to his pursuit of understanding.

c) Cliff uses the model – on 17/11 and on morning 19/11

Exhibit 6 – Continuation of exhibit 5; discussion of 17/11

Short discussion about impact of likely restructuring on Bank and possibility of selling

C And we don't have And in terms of our conclusions, **we are happy with their carrying value where it is relative to the market value?**
E **Um yeah,** I think it is basically because they use such a rough sort of approach. Well you know that they are not intending to exit at all. So they just use it as a guide rather than.
C Yup.
E **Well you know it seems reasonable, it is not err wildly......**
C OK.
C In terms of where that provision has been relative to the market value in the past.
E Um.
C Are we consistent with what we have done last year, for instance?
E Um...um...I'd have to look back in the files to understand. I remember the numbers were roughly the same. *(Eric turns pages in file and he and Cliff look at page 2 of the half year credit update).*
C Yeah So we are not fiddling.....
E *(Confident authoritative tone)* It was quite small. There was £2m difference last time.
C OK.

Eric tells Cliff about one or two commercial developments that have occurred since the half year. Cliff raises the need to consider US GAAP; Eric says that this is to done by the group audit team at the year end. Cliff moves onto next loan.

Here Cliff was essentially asking Eric if he was "happy" with the current carrying value of the debt *vis a vis* the market value. Although Eric said he was (beginning of line 5) giving two reasons (lines 5 to 7), it is clear from his follow up comments, "Well you know it seems reasonable, it is not err wildly....", he was beginning to have some reservations. Cliff did not express his view at this stage. Instead he asked Eric about the past position. As Eric did not remember the exact position he turned to the half year paper and noted that the difference was "£2m....last time" (line 17). He did not comment on whether or not the carrying value was higher than the market value. This was not necessary because both were looking at the relevant table. In this exchange Cliff used the valuation model to improve his, and indirectly Eric's, understanding of what the current position was by asking Eric for his opinion and by setting up a comparison of the results for the current period with that for the half year.

Cliff had a short meeting with Eric (19/11 meeting) after he had reviewed Eric's files to discuss his review points. Cliff had one point on Kookaburra. He started the discussion by referring to the £9 million difference between the carrying and market values and noting that the Bank had sold its Equity notes in Kookaburra and had taken the profit, which was equal to the proceeds, straight to the profit and loss account. (Details in footnote to the Bank valuation schedule.) Cliff then emphasised what had changed in the quarter; the market values relative to the carrying values had fallen and "we also seem to have sold those Equity notes and released the full amount." He asked Eric to check out situation with the client and suggested that Kookaburra should be dealt with in the Audit Committee Paper.

Workplace mathematics In these two episodes Cliff used the model in three different ways, each involving *workplace mathematics.* In the first episode, he used the current result produced by the model to get Eric to express his opinion – to express his reservations about the current provision, and he also used the results from the current and half year valuations to generate a comparison. In the second episode he effectively modified the model, by suggesting the comparison between the Bank's exposure at the end of the second and third quarters should take account of the impact of the sale of the Equity notes in the third quarter.

I also believe that Cliff was indicating that he was concerned about the adequacy of the current provision, despite having written the note "Provision level seems adequate" on Eric's credit update paper. (Evidence: suggestion that Kookaburra should be mentioned in the Audit Committee Paper and his comments to Tom (below).)

d) Tom understands and uses the model – late afternoon on 19/11

This was Tom's first review meeting relating to the loan loss review. Cliff and Eric specifically raised Kookaburra as of being of particular concern. Cliff reported that the Kookaburra Equity notes had been sold and the profit had been taken straight to the profit and loss account while "the market values of the debt versus the carrying values had gone slightly the other way." Tom noted the comments but delayed considering them until he turned to the Kookaburra file and asked for an update. The team's discussion is described at length because the usage of *workplace mathematics* is interwoven as part of the commercial considerations.

Eric reported that the Bank's exposure *(quantum of the loans outstanding) was broadly the same as at the half year*, and that the bad debt provision and the basis on which it is calculated had not altered. Tom asked *"How much is the market value of the debt?"* Eric turned to the valuation, showed it to Tom and said *rounding and reading the numbers from the schedule "77 and we are holding it at 86.... so there's 8"* (8.55m in fact). Tom *looking at the valuation said "Right"*, and Eric added "Well at the half year there was only ...2 million difference." Eric referred to the recent sale of the Equity notes and Tom acknowledged that in the past the ownership of the Equity notes provided some comfort with respect to the bad debt provision[1].

The team now had a shared understanding of the current position.

[1] *Workplace mathematics* in italics.

Tom obtained an understanding of the position more quickly than Cliff, partly because he has been through the position many times before but also because Cliff and Eric were in agreement on the key facts and briefed him succinctly. Eric had also learnt from his earlier briefing of Cliff; he directed Tom to the key facts immediately, "77 and we are holding it at 86.... so there's 8". Eric showed improved cognitive efficiency.

Tom's take on the position was quite different from Cliff's and Eric's. He explored the validity of using the valuation model to assess the bad debt provision. He started by identifying that there are two issues: the inevitable future restructuring of the Kookaburra business and "where we are today". Tom asked Eric if he had looked at the underlying company position. Eric had not. Tom pointed out that, as the Bank was not intending to exit its relationship with Kookaburra, using market prices as a guide to the provision might be "spurious". He therefore suggested that looking at possible future operating cash flows for the underlying business might provide evidence justifying the use of market prices. Tom and the team looked very briefly at a summary of recent operating performance, and the team concurred with Tom that the company would not be able to service and repay all the debt. After further discussion Tom concluded that, with all the uncertainties surrounding the Kookaburra business, any estimate of the likely write off would be "a finger in the air" figure. Cliff pointed out that theoretically the market prices should take account of the Market's views about future cash flows and liquidity. Tom countered that market prices may be affected by factors which operate independently of the underlying business, so that there could be reasons why a valuation based upon the secondary market values might not be valid. Cliff suggested that the same might be true of trying to best guess the restructuring. Tom agreed but added that if you looked at both of them, then hopefully they would not be "hugely" inconsistent. It was agreed that the loan would be raised in the Audit Committee Paper and Tom wrote on his queries list "Refresh Kookaburra logic check on balance sheet" (meaning model future estimated cash flows roughly) and then moved onto the next loan.

On 27/11 Tom carried out a detailed review of the Kookaburra file. He started by reviewing the accounts and doing a back of the envelope calculation of the sustainable debt using a cash flow modelling approach (for detailed discussion see section 8.3). As a result of his modelling and reasoning (which was substantially mathematical) he concluded that the market prices broadly reflect the amount of debt that was then sustainable by the business. He then briefly reviewed the report of current business operations; there had been little change. He looked at the Bank's valuation of the debt in the working papers file and then read the current and prior periods credit update reviews. He concluded that there were so many uncertainties surrounding the business that the only approach was to use that adopted by the Bank, namely to look at the securities and their values in the secondary market. As he worked he commented to me "I am not unhappy with the provision they have come up with". He, however, noted on his queries list that if the Bank used the value of the security as the basis for establishing the carrying value of the loan, the Bank should increase the provision by a further £8.55 million from a US GAAP perspective, i.e., in the US accounts of the group. (The Bank reports results under both UK and US GAAP.) Tom also decided that he would not ask Eric to estimate how much debt might be written off if a restructuring were to be carried out at the year end; there were so many uncertainties and the reality was that the audit team would conclude that the appropriate carrying value for the debt should be based upon a valuation using market prices.

Here Tom's main task was to assess how the Kookaburra bad debt position should be calculated. He explored the validity of using the valuation model and, after discussion and consideration of the possible future cash flows, concluded that the only approach open to the Bank was to use the model and that further provision was needed in the US GAAP accounts. On the basis of the evidence collected I consider that at this stage, Tom probably had not made up his mind about the UK accounts position.

Validating the use of the model was Tom's main contribution to the cognitive work of the team. The consideration of this was essentially an accounting matter. References to evidence to substantiate validity include references to hypothetical results generated by other models – cash flows, market prices and operating results, all of which have substantial mathematical characteristics. However commercial considerations predominated. This vignette also illustrates clearly the sharing of cognitive effort within a team. Tom raised and clarified the issue in a meeting. He, however, completed the assessment of its validity working alone. He then effectively shared his conclusions with Eric and Cliff when he crossed off the request for a cash flow analysis from his list of outstanding review points and added the US GAAP point. In subsection 8.2.6, I discuss at length whether or not assessing the commercial validity of a model and acting upon its results are part of *workplace mathematics.*

e) The team changes its view in drafting meeting – on 9/12

On 9/12, the team had a meeting to review the first draft of the Audit Committee Paper. Tom redrafted parts of the paper and indicated to Cliff and the rest of the team how he would like it to be amended further.

Exhibit 7 – Extract from annotated copy of first draft of Audit Committee Paper

<u>Kookaburra</u> - ....The carrying value of the junior debt (after taking into account the provision) is currently £86 million. The market value of this debt at 2 December was £78 million (31 December 2002 £87 million). The Bank continues to review the level of the provision against the market values for the junior debt[,] ~~as management consider they provide the most reliable~~ *a* guide to ultimate recovery. Given the minor nature of developments in 2003, ~~we consider the provision~~ [level to be appropriate]
and the fact that
security [approx.] = S[econ]dary
mkt price ~~*etc.*~~ *[provision]*
looks ~~*reasonable*~~
underprovided

Text in [] added to clarify meaning

This part of the paper was drafted by Eric and amended by Cliff prior to the meeting. The annotations on the text (all the crossings out and the text in red) were made by Tom during the meeting. The paper uses the results from the valuation model to compare the current position with that at December 2002 (not the half year as the report to the Audit Committee is a report on the annual audit).

Tom read the Kookaburra paragraph. Cliff told Tom that "we (meaning Cliff and Eric) were quite specifically tying the provision level to the secondary market price[s]" while management "have calculated the provision based upon market value of the general debt" as they considered market prices to be a reliable guide. Later in the conversation Eric confirmed that this was his understanding of management practice. As Tom was considering the drafting, he remarked that the inclusion in the draft of the text in the black square brackets, which indicated that the audit view has not yet been finalised, was correct. As the discussion and redrafting progressed he concurred with Cliff's view that the audit team's view of the provision had moved from considering the provision to be prudent to being reasonable because the market values were down and the equity had gone. He first wrote *".....[provision] looks reasonable"* but then he reconsidered saying "now it may be" and changed *"looks reasonable"* to "***underprovided***". Cliff and Eric do not challenge his conclusion; they effectively concur. Tom then moved on to other matters.

Workplace mathematics Thus the team's audit opinion emerged unspoken *"............[provision] looks ~~reasonable~~ underprovided"*. It was derived from consideration of the results of the debt valuation model.

f) Resolution

On 22/12 during his second review of Eric's working papers Cliff told me that he was not going to carry out his final review of Eric's file on Kookaburra as he and Tom had raised the issue of a possible underprovision with the Head of Credit and the Bank's management were reconsidering the position. Cliff subsequently told me that the Bank had decided to mark the debt to market. The final Audit Committee Paper issued on 19th January 2004 reflected this position, namely that the debt is carried in the 31st December balance sheet at market value using the secondary market prices current at that date.

8.2.6 Discussion

Here I consider three things: how the team used the model as they formed their audit judgement; *workplace mathematics* including whether or not assessing the validity of the model and action in the world are *workplace mathematics;* and some of the ways in which cognitive effort was shared/distributed.

a) Auditing practice summarized (In this paragraph tasks undertaken are categorized and numbered to aid the subsequent discussion.)

The debt valuation model and the commercial meanings derived from its results became the pivot for much of the team's work on the Kookaburra loans. All key discussions were focused around the valuation. The team understood how it was used by the Bank; Eric explained to Cliff how the Bank used it and both jointly explained to Tom (1). Cliff and Tom sought and gained understanding of how the model worked and the results it produced; Eric assisted them to navigate and understand the papers (2). Cliff modified the model in conversation by bringing the implications of the sale of the Equity notes to the fore (3). The team compared the current results to those produced by the model at the half year and at the previous year end (4). Tom validated the use of the model (5). In turn each member of the team expressed reservations about the current bad debt provision; those reservations were directly linked to the results produced by the model (6). Finally the gap between the market and carrying values shaped their audit judgement, namely that the Bank's bad debt provision in early December was underprovided (7). Much of the work done by the team was done in conversation and discussion, and not preserved in audit records.

b) Usage of workplace mathematics (That is, other than that of a computational nature – dealt with in chapters 7 and 5)

The model is both a mathematical and an accounting model. As is described above much of the work done incorporates *workplace mathematics*. The episodes marked (1) (2) and (4) clearly do so (Evidence in *workplace mathematics* paragraphs: for (1) – 8.2.5 a) and d); for (2) – 8.2.5 b) and d); and for (4) – 8.2.5 c) and d).) and that work is significant as it provided the foundations for the formation of the audit opinion. Each member of the team expressed concern about the bad debt provision; these actions (6) were commercial interpretations of the model results and as such are not *workplace mathematics.* However in each case that concern was derived partly from mathematical relationships, the fact that the carrying value of the debt exceeded its market value by £8m or £9m and a comparison with a past position. The same is true of the emergence of the audit opinion in the drafting meeting (7). I consider that Tom (and the team) used *workplace mathematics* as they downgraded provision from 'prudent' to 'looks reasonable' to 'underprovided'; in each successive assessment Tom was re-evaluating the impact of the £9m difference and possibly the sale of the Equity notes.

Episodes (3) and (5) are discussions that are primarily commercial in nature. This is particularly true of the validation discussion. The episodes as reported here (i.e., excluding Tom's consideration of the possible future cash flow position which is dealt with in detail in section 8.3) do not show the team using much *workplace mathematics,* but some was used. Cliff in bringing the impact of the sale of the Equity notes to the fore influenced the team's interpretation of the £9m difference between the carrying and market values. Two mathematical models – a cash flow model of possible future operating cash flows and market price valuations for securities – underpin the validity discussion. This suggests further complexity about the nature of the usage of *workplace mathematics.* A discussion about the commercial implications and interpretations of mathematical models and their results presuppose some understanding of the mathematical nature of the models; this is certainly evident in the discussion between Tom and Cliff on the validity of using market prices as a guide in calculating the bad debt provision. Also when the team determined that Kookaburra was not currently able to service its debt through a cursory review of the operating results they used *workplace mathematics;* they derived their conclusion from a consideration of Kookaburra's operating results. *Workplace mathematics* used thus formed part of the discussions. However, the discussions were primarily commercial, not mathematical.

On the other hand valuing the debt at 31st December 2003 prices was and then actually increasing the bad debt provision and so decreasing the value of the debt to that value in the accounts and the accounting records was *workplace mathematics;* it was *workplace mathematics* realised in action (the action was primarily performed by Bank staff).

c) Distributing cognition and workplace mathematics

The use of texts and audit processes distributed necessary cognitive effort over time. Texts were used to save individual effort (e.g., the Bank valuation schedule), to record results of work to date (e.g., Eric's working paper and lists of review queries) and to re-focus and progress discussions.

Cognitive effort was shared between team members in a number of different ways. All team members worked to understand how the model worked and the results it produced. Through discussion they ensured that they shared common understandings. Eric aided Cliff and Tom as they strove to understand. Also all came to express concerns about the current provision. On the other hand effort was shared as parts of the task in hand were carried out by one member of the team only:

- Eric drafted the working paper;
- Cliff surfaced the sale of the Equity Notes;
- Tom alone estimated the amount of the loans that Kookaburra might have to write off if it were to restructure at, say, the year end (see section 8.3 below); and
- Eric and Cliff jointly wrote the first draft of the Audit Committee Paper.

Other team members accepted much of this work without questioning it.

Effort was shared in another way. As Eric took Cliff through the model (see exhibit 5) they seemed to share the effort of understanding how the model worked and what it showed, with the balance of effort switching from Eric to Cliff as they worked through the model. There is also a similar sharing of effort evident in the way the view about the provision emerged. Refer to exhibit 7. The opinion in the unaltered draft paper "we consider the provision [level to be appropriate]" indicated that the audit opinion had not yet been finalised. Tom first agreed with Cliff that the opinion had moved from 'prudent' to 'reasonable' but in the course of redrafting moved it further – to 'underprovided'. Thus it seems that sharing of cognitive effort allowed the team to develop and share common understandings of the position and, perhaps more importantly, allowed an effective and thorough re-evaluation of the provision to evolve with each team member being able to contribute new thoughts as the audit process unfolded. Sharing cognitive effort in an organised manner can in itself be a powerful tool.

Thus cognitive effort including the embedded *workplace mathematics* cannot be separated from the contexts in which the work was done, particularly when one takes account of past cognitive efforts; they are both part of the context and what is done. The task in hand turned out to be the re-evaluation of, rather than the evaluation of, the Kookaburra bad debt provision. It was carried out during the normal manager and partner audit review process and the drafting of the Audit Committee Paper. Team members, both individually and collectively, brought knowledge and skills to both the consideration and the resolution of the audit issue; the knowledge and skills were mainly those of accounting and auditing, the particular facts and circumstances relating to the case in question but also included proficiency in understanding, using and creating texts and mathematics, particularly relevant *workplace mathematics*. Key institutional resources here included the audit process and practice, audit working papers, and accounting practices. All these taken together substantially structured the re-evaluation of the bad debt provision. In particular texts referred to during that process – Eric's credit update paper, the Bank's valuation schedule and the draft of the Audit Committee Paper – frequently initiated and to a certain extent structured the discussions that led to forming the audit judgement. However it was the debt valuation model, a mathematical model, that was the pivot of the discussions.

8.3 Kookaburra cash flow model – an ephemeral model which was used as constructed

8.3.1 Introduction

Cash flow models are much used accounting tools. They are also mathematical models. The accounts of UK quoted companies all have a cash flow statement and any company which is raising money through an equity issue will forecast its future cash flow requirements for a suitable period, often eighteen months to two years after the issue. The purpose of the two statements is different. The first shows how the cash and other financing generated in a year has been used while the second can be used to predict whether or not the company needs to have overdraft facilities in place over the forecast period. A complete cash flow calculates the cash generated from business operations and reconciles it to the year end cash balance taking account of changes in financing and payments to (and receipts from) shareholders. In practice, it is often only appropriate to model part of a cash flow. In this and section 8.4, I show how Tom and Eric constructed and used cash flows to assist them with their audit work. Eric constructed his to show what had happened while Tom's was used to reason about likely future events. Where I am reporting on work observed I use the present tense as the description is derived directly from my observation notes (and the documents used or constructed).

I now analyse in detail how Tom assessed how much debt Kookaburra might have write off if it were to be reconstructed. We get an insight into Tom's thinking because he talked to me as he reviewed Eric's Kookaburra file. First I put the ten minute episode into context, second I assess Tom's professional expertise, third I analyse the episode in depth emphasising the usage of *workplace mathematics* and then I conclude by foregrounding a limited number of themes. Here we see an expert at work; using cash flow analysis, Tom creates a logical argument to estimate the quantum of a possible write off if the 2002/3 cash flow position were maintained.

8.3.2 Background

In the extract below we see a virtuoso extempore performance. Although I am the audience, the performance is not primarily for me; it is for Tom, for Tom himself. Tom uses the 2003 interim results of Kookaburra to calculate approximately how much debt it might have to write off if it were to undergo a reconstruction. The decision to do the calculation is informed by Tom's preliminary conclusions on Kookaburra and the results of this exercise inform both his assessment of the bad debt provision and of the additional work Eric should undertake. In the preliminary review meeting Tom had decided that as the Bank's current intention was to hold the debt, the team should look at both the secondary market prices and what might happen on a reconstruction (subsection 8.2.5 c)). In about ten minutes Tom determines how much debt Kookaburra might have to write off – approximately 25%. He immediately assesses that "the market prices aren't as far away as I thought". He then reads Eric's working papers file fairly carefully, and decides that the Bank approach of using the secondary market values to assess the value of debt is the only sensible approach. In fact, Tom uses his calculation (with other facts) to draw four conclusions: one, valuations of the debt using either market prices or an estimate of the likely write off are likely to produce similar answers; two, the uncertainties attaching to the future operation of the business are so great that using the secondary market prices to value the debt is the only sensible approach; three, from a US GAAP perspective the Bank will have to mark the Kookaburra debt to market; and four, as the uncertainties surrounding the operations are so great it is not worth Eric spending time producing a working paper estimating the likely write off in a future reconstruction.

Exhibit 8 is part of my record of Tom's review of Kookaburra. The details in lines 10 to 50 need to be read together with the discussion in subsections 8.3.4 b), c) and d).

Exhibit 8 – Extract from my observation notes; Tom's review of Kookaburra file

Headnotes:
Text in ( ) added to aid reading
Text (…. *) – my speculations

Tom takes all copies of accounts relating to loans to be reviewed from the accounts file and puts them on the table on his left hand side. Has Eric's working papers file, blue note book and calculator on table.

3.00pm Starts **Kookaburra** review.
Picks up 2003 interim accounts and looks at them.
"I am looking at the balance sheet and profit and loss indicators.
"They are all kind of [what] you would expect – bucket loads of fixed assets."
He has his fingers under the balance sheet figures.
"Unhelpfully, this doesn't tell me what the total debt is."
Turns to the financial analysis in the interim accounts.
Reads the operating results and the notes by its side. *(The results are in a columnar format)*
Turns to next page and focuses on 'summary combined account'.
"It is not clear……..

"Total creditors of	£7.4bn	
"Core debt	4.8	
"Buffer debt	1.5	

"That's	6.3	long term debt
	7.4	creditors
"Assumption is	1.1	current liabilities

"Not entirely stupid.
"So not pursue.
"So we have got 6.5bn debt and can compare that *(interest thereon with finance charges)*."

Punches 6.5 * 0.05 into calculator.
"That's approximately 350m to 400m interest every year. It might be less because rate is sort of proxy for sterling but the euro rate is lower.
"Financial charges £342m for last year. *(Net charges are £320m – from accounts.)*
"Therefore… building up from that. £320m net *(of financial income)* plus £386[m] of operating expenses…. we need turnover of £700m at least before we can repay debt.
"Cash flow would be much higher [than resulting profit] because of depreciation."
Uses calculator to obtain net cash flow – takes numbers from statements (effectively adding back depreciation) *(194 +140 = 334)*
"Cash flow is paying the interest but not contributing to repaying capital"

Goes to A5 blue note book and writes exploratory equation

"x/90 +x*0.05 = 350"
"I am trying to get a feel of where turnover needs to go if we are to repay debt or alternatively if we are to write off to put on an even footing."

Does an exploratory calculation on .. calculator *(using equation. I did not see numbers used.)* ***(3.25/90+3.25*0.05=0.199=199m<£350m or near*)***
Hesitates.
3.10pm "I thought they needed to half the debt to make it".
More calculations on calculator: 350*1/4= 87.5, *(rounded down to 85)*; and 85*83=7033.
(Close to 6.3bn – so can repay 75% of debt within time frame of concession.*)
"So we might have to write off 25%. Maybe the market prices aren't as far away as I thought."
…………..
Reads Eric's paper and working papers file fairly carefully.
"There are a number of negatives. I'd forgotten about the minimum usage charge ………….. There is a guaranteed minimum payment. The [usage] is lower than expected therefore the charge is likely to be reduced when [the] agreement [is] renegotiated …….
"Therefore what am I going to do
"The Bank look at the security and the value of the debt in the market. This is the only sensible approach as there is too much uncertainty because of the minimum usage charge and different levels of debt will be affected differently by any shakeout. I am not unhappy with the provision they have come up with ."
Reads Eric's conclusions.
" They have just gone on market value and the fact is that little has changed…
"We could do calculation to work how much debt we should write off – it would be a fag packet calculation. The reality is that we would come back to market value at the end of the day
3.25pm "*(I will not)* ask Eric to do……"

8.3.3 Tom, an expert at work

How does one unpick this episode to provide some understanding of the expertise demonstrated here. Because Tom's argument and calculations are deceptively simple it is easy to underestimate his expertise. The focus of this thesis is *workplace mathematics* in action. I therefore analyse in depth Tom's mathematical practices. However I start by evaluating the nature and depth of Tom's professional expertise as an accountant and auditor.

Tom is an audit partner. He is a qualified accountant with substantial auditing experience of banking institutions. He is a good team leader (observation and comments from colleagues who are known to both of us), who willingly shares his expertise with team members. Using the expert/novice paradigm, Tom is an expert. In the context of the assessment of the Kookaburra loans, particularly estimating the order of size of a possible write off, Tom brought substantial knowledge and skills to the task:

- he has good knowledge and understanding of financial accounts;
- he has conducted many loan loss reviews as an in charge, manager and partner. He is, thus, both experienced and practised;
- he has considerable experience of understanding and using cash flow analyses – see Possum (section 8.4), his tutoring of Eric on Wallaby (section 10.7) and below;
- he knows a lot about the Bank's Kookaburra loan exposure as he had looked at it before on previous half year and year end audits – more than once;

- he has a well developed framework for analysing the assessment of bad debt provisions – outlined to me at the beginning of his review of Eric's files and often alluded to in his preliminary review meeting;
- as Tom reviewed the Kookaburra accounts, papers and files he not only sought to understand the facts, he extracted what he saw as key facts (not always those highlighted by Eric and Cliff, e.g., Wallaby and Koala) and placed meanings on them that assisted him to make judgements about the Bank's bad debt provisions. He knew what he was looking for – observed;
- he focused on the big picture as he developed his understanding of positions – often observed recapping key points/issues; and
- in conversation in review meetings he continually melded distinguished key facts into arguments that led to the making of audit judgements.

Based on observation, Tom's framework for auditing loan loss provisions was well grounded both in experience and theoretically, and was to a certain extent a personal construct. It gave him the facility to gather key facts, identify missing information, and reason and evaluate a situation simultaneously, rather than consecutively. Tom demonstrated all these skills as he estimated what the debt write off might be if Kookaburra were reconstructed. His skill in focusing on the big picture and key issues and drawing reasoned conclusions not only related to accounting and auditing as such but was also reflected in his extensive and flexible use of *workplace mathematics*. Tom's performance was effective and efficient, and fit for purpose.

8.3.4 What Tom does analysed

a) Introduction

At the time Tom starts his detailed review of Eric's Kookaburra file, final assessment of the Kookaburra bad debt provision is still outstanding. As Tom looks at the 2003 interim results, the latest which have been published, he focuses upon the long term debt almost immediately, saying "Unhelpfully this doesn't tell me what the total debt is." He doesn't tell me what he is doing. Looking at what he did, he was clearly answering two questions: 'What is the annual cost of debt service including repayment?' and 'Given that the cash flows from the business cannot sustain current levels of debt, what proportion of the debt is likely to have to be written off?' I think it is reasonable to postulate that he does not say what he is doing because it was self evident to him. Furthermore, the path that Tom lays out is driven by the answer the second question, which he initially believes to be the order of 50%. Tom's expertise is such that he was able to structure what he did without conscious thought or planning; the evidence for this is the almost uninterrupted flow of his argument and the fact that he only reveals what he is about when he gets answers, e.g., "Cash flow is paying the interest but not contributing to the capital repayment" and "So we might have to write off 25%."

Cash flow analysis – essentially a mathematical model – provides an overarching framework for Tom's analysis and modelling. Initially when I analysed this episode, I thought that two models might underpin Tom's analysis – a cash flow model and a breakeven model. I subsequently revised my view as I believe that Tom's comments during the preliminary review provide powerful evidence to the contrary. When he was looking at the Kookaburra file, he identified the need to look at the Kookaburra cash flows and when he was summing up right at the end of the meeting he said:

"..you have done the right things in terms of looking at gradings and what's happening in the market and what's happened recently. Can we just make sure that on each [case] ...... we understand where the money is coming from in terms of the underlying business.........Everything else is built [up]on that .... Where we are relying on [the market value of the] security but we know that we are actually not going to sell ......we ...need to know that those prices are not just fantasy. Does that make sense?"

From a research perspective, Tom's cash flow analysis can be broken down into three phases: estimating the long term debt and the interest thereon; estimating the cash flow available to service the debt; and estimating the likely debt write off. I discuss each in detail.

b) Estimating debt and interest thereon

The first phase comprises estimating the quantum of the long term debt and the annual interest thereon (exhibit 8, lines 10-29). Tom makes these estimates as he looks at and extracts information from the summary accounts and financial analysis section of Kookaburra's 2003 interim results. Tom looks at the numbers in the balance sheet and notes that the balance does not itemize the total interest bearing debt (it is included in the creditors figure). He rounds and memorizes the creditors figures as "total creditors £7.4 billion", as evidenced by his subsequent use; the accounts figures are in this form except they are not rounded. He turns back a page to the financial analysis and reads the table showing the operating results and the note on financing. As he reads he says, "It is not clear.......", and then articulates his understanding of the position: "Total creditors 7.4 billion pounds (from memory). Core debt 4.8 (from financing note). Buffer debt 1.5 (from financing note). That's 6.3 long term debt (adds the two numbers in his head and attaches meaning to the result – long term debt). 7.4 creditors (restates presumably to set up next mental calculation) Assumption is 1.1 current liabilities (performs subtraction mentally and interprets meaning as he does calculation). Not entirely stupid. So I'll not pursue." Tom concludes, "So we have got 6.5 billion debt...", rounding to the nearest half billion.

Workplace mathematics What has Tom done here? He has calculated the long term interest bearing debt using information from the interim accounts.. He could have done that by just adding the core and buffer debt numbers. He does much more. He situates the estimate within its accounts category – the total creditors, a definitive figure he is happy to rely upon. He thus calculates and verifies at the same time; he integrates the mathematical, accounting and auditing practices as he goes along. As he sets up and does the calculations, he retains or assigns a commercial meaning to each number as it is used or created. He thus holds both the mathematical and commercial meanings in his working memory. He does not move from the world of commercial meanings to the world of mathematical modelling and back to the world of commercial meanings. He works in both worlds simultaneously with the world of commercial meaning having precedence. From a mathematical viewpoint it is worth noting that throughout this exercise, Tom operates with numbers (in this case billions to one decimal place) that are easy to manipulate mentally. Expertise is demonstrated here through the integration of practices and the ability to see big picture.

Tom moves on, "So we have got 6.5 billion debt and [we] can compare [interest]..." By using the word 'compare' Tom is signalling that he is going to test a hypothesis, namely that the notional interest on 6.5 billion, is broadly similar to the interest cost in the accounts. Tom does not say this but carries out the comparison. He punches "6.5*0.05" into his calculator (i.e., annual interest of 5% on 6.5 billion). The answer is 0.325, which he uses saying "That's approximately 350 million to 400 million interest every year." He justifies his use of 5% by reference to UK and Euro base rates, suggesting the rate might be high. He then checks his estimate against the figures in the summary accounts, "[The] financial charges were £342 million for last year," and signals he is satisfied by proceeding, "Therefore..."

Workplace mathematics By setting up the comparison, Tom is both calculating an estimate of the annual interest charge and testing its reasonableness at one and the same time, and as he works he attaches commercial meanings/justifications to all his numbers. There are differences between these calculations and those above. Here he works from his model, 6.5 billion of long term debt, to information in the financial accounts, not from accounts information; and he punches the calculation into his calculator (possibly because it is more complicated in that it involves multiplication and place values). Tom's rounding practices are noteworthy. 6.3 billion is rounded appropriately to the nearest half billion. He is, however sometimes, cavalier with the normal mathematical rules for rounding; 0.325 billion is read as 350 million and used as the lower bound for a range with an upper limit of 400 million. Note that in the process Tom converts units from £ billions to £ millions. Tom works with broad approximations, not precise mathematical ones as he is only seeking a broad estimate of the debt write off as is demonstrated later. It is arguable that the way he rounds up reflects a basic tenet of accounting, namely erring on the side of caution.

Phase one is complete. Now that Tom knows the quantum of the long term debt and the annual interest burden, he seeks to establish the extent to which the cash flows can service the debt. Tom already knows that the company's operations cannot support the current level of debt and he believes that about half may have to be written off – he specifically referred to this fraction in his earlier discussions with Cliff and Eric. These understandings influence Tom's approach in phases two and three.

c) Estimating available cash flow

Tom's next step is to estimate the cash flow available for debt service and in that process shows that they cannot service the existing debt burden (exhibit 8, lines 30-35). Using the 2002 results in the 2003 interim results (used because the 2003 results are only for the half year), Tom proceeds "Therefore building up from that £320 million *(pointing to the financial charges (£342m) and income (£22m) in the 2002 summary profit and loss account)* plus 386 of operating expenses *(from same account)* we need turnover of at least £700 million before we can repay debt." In the course of this commentary Tom carries out two calculations in his head: 342 – 22 = 320 being the difference between the financial charges and income; and estimates 320 + 386 to be 700. Tom notes that the cash flow would be higher [than profit?] because of depreciation. Kookaburra's turnover in 2002 was £581m, way short of £700m; Tom does not comment on this but must have noted it as the figure is immediately above the operating expenses figure he uses.

Workplace mathematics Again as information is extracted from the accounts relatively simple calculations are performed mentally and commercial meanings are attached to all the key numbers. From a mathematical modelling perspective, Tom uses a very simple cash flow schema to estimate the cash the business needs to produce to service the debt interest. Tom knows that the business cannot sustain the existing level of debt and so, does not refine calculation.

He immediately moves onto using another cash flow schema to establish an estimate of the annual cash flow available to service the debt. He calculates this on his calculator. He adds an estimate of annual depreciation obtained from the accounts to operating profit for 2002 and comments, "Cash flow is paying the interest but not contributing to repaying the capital." Generally depreciation is the main non cash item which is deducted in arriving at a year's profit. The numbers were punched into the calculator too quickly for me to observe. A possible reconstruction is £194m (operating profit for 2002) + £140m (2* half year depreciation charge) = £334m. This is close to the accounts total finance charges of £342m, which would enable such a rapid conclusion (exhibit 8, lines 33-35).

Workplace mathematics Tom uses two very simple models to evaluate Kookaburra current situation *vis a vis* debt service. This is acceptable as Tom is taking a broad brush approach. With the first he demonstrates that Kookaburra needs turnover of at least £700 million to sustain the debt. With a second equally simple model he estimates the annual cash flow available for debt service. We see here effective flexible use of the cash flow modelling framework – two different schema being used in quick succession. Each illustrates a different point, so moving on Tom's argument.

d) Estimating debt write off

Tom's interim conclusion confirms his prior understanding that in due course Kookaburra will have to write off part of its debt in a reconstruction. He then proceeds to estimate it (exhibit 8, lines 35-50). Tom goes to his A5 blue note book, which he uses for notes and queries relating to the Bank audit, and writes down the following algebraic equation

$$x / 90 \quad + \quad x * 0.05 \quad = \quad 350$$

and then says "I am now trying to get a feel of where turnover *(sic* means cash flow) needs to go if we are to repay debt or alternatively [what] we are to write off if we are to put on an even footing", essentially setting out the rationale for the exploration of the cash flow just undertaken. The 350 here represents the cash flow (i.e., £334 million rounded up). He has already answered the first of his original propositions and the equation's purpose is to deal with the second.

Workplace mathematics What does the equation represent/do? Its purpose is to enable Tom to estimate very broadly the level of debt that can be sustained in the short to medium term (5-10 years) assuming current levels of cash flow – estimated to be £350m. 'x' represents the long term debt, 'x/90' an estimate of the annual repayments of loan principal and 'x*0.05' the annual interest charge in the short to medium term. 90 is an approximation for 83, the unexpired term of Kookaburra's operating concession.

Tom does not solve the equation mathematically. He already has expectations about how much Kookaburra should write off; on two occasions I have heard him express the view that Kookaburra will have to write off half the debt. So he tests this outcome on his calculator using the left hand side of his equation. I did not see the numbers he punched into his calculator but presume them to be as follows, 3.25/90 (= £36m capital repayment) + 3.25*0.05. This sum equals 199 which is approximately 150 less than 350. This implies that £150 million free cash flows would be left after debt service – banks would probably not leave this amount free on a reconstruction. The answer surprises Tom. He hesitates for a short period. After hesitating he says "I thought that they needed to half the debt to make it".

During this period he decides that a 25% write off may be more appropriate but does not say so until he has done two further calculations. I believe he may have reasoned along the following lines; as interest on debt uses approximately 100% of the cash flow, namely £350m, and £350m is also an estimate of the current interest obligation, interest on 75% of debt will use 75% of the cash flow, leaving 25% for capital repayments in the event of a 25% write off. He proceeds to do some more calculations on his calculator using a completely different approach to explore the 25% hypothesis. He sets out to test whether Kookaburra could repay the rest of the debt if 25% were written off. He calculates one quarter of the available cash flow 350*1/4 = 87.5. This gives an estimate of a possible annual capital repayment. Tom rounds down to 85. He then punches 85*83 into the calculator being an estimate of the repayments that could be made during remaining term of the concession; this equates to £7033 billion, or approximately £7 billion, which means the reconstructed debt (75% of £6.3 billion) could be repaid (in about 60 years – my estimate). Without further calculation or modelling Tom concludes "so we might have to write off 25%. Maybe prices aren't as far a way as I thought." Tom never explains why he decided that 25% was the right figure. In retrospect, I think that he was slightly put out by the 50% answer and so he said less as he adjusted his expectations.

Workplace mathematics – nature of performance How Tom constructs and uses the repayment model – an algebraic equation – is distinctive and of interest. He never solves the equation. It is used as a framework for thought. The model is structured by and used to structure and support Tom's thoughts and argument. Tom wishes to estimate in broad terms the level of capital repayment and interest that could be sustained by a cash flow of £350 million. By writing the equation down, Tom both anchors his thoughts outside short term memory and sets out the logic of his argument for himself and for me. The equation itself is a mathematical representation of the problem Tom is seeking to answer. He does not solve it because he believes he knows the answer, i.e., approximately 50% of the debt needs to be written off, and so he tests that supposition in the formula. The result demonstrates to Tom (to his surprise) that his belief is incorrect. He still does not solve the equation. He considers the answer to the calculation and revises his expectations, taking a little time. He decides that 25% may be a better estimate. He may have remembered that the Kookaburra provision was of the order of 25% or that that three junior debt prices lay between 50+% and 89% of nominal value. His comment when he finishes the exercise, "Maybe the market prices aren't as far away as I thought", indicates that the debt prices may have had some bearing on his new estimate. Also there may have been some element of "if not 50%, then 25%." Tom then tests his new hypothesis using a different model. He checks that Kookaburra would be able to repay the reconstructed debt before its concession expires if it were repaid at the rate of £85 million per annum. Implicit in this test is the assumption that 25% of the cash flow is released for repayments if 25% of the debt is written off. By using a different model to support his reasoning, Tom strengthens his argument and his conclusions. It is however to be noted that he never checks his answer by substituting the new numbers back in a variation of the left hand side of the equation; 0.085 + 6.5*75%*0.05 = 0.329, that is £329 million.

Workplace mathematics – modelling Here Tom constructs and uses two ephemeral models; they are used to test hypotheses. The first, **which is expressed as an algebraic equation,** is created, written down and then used on a trial and error basis, leading to an unarticulated adjustment of expectations from 50% to 25% for the write off. The second is expressed in two calculations and provides support for the new hypothesis. The content and, to a certain extent the structure, of both models are governed by the particular facts and circumstances of the Kookaburra business and are simple, uncomplicated theoretical constructs. The usage of the first model is driven by an expected answer. When Tom's initial expectations are not borne out, he accepts the results produced by the model, reviews the position and alters his expectations. The second model is used to test the new hypothesis, namely that if there is a 25% write off of debt and 25% of the calculated cash flow is used for capital repayments, Kookaburra will be able to repay the principal well before its concession expires. Tom has the confidence to use simple models and to accept their results, even in circumstances where his expectations are confounded. This confidence is probably the product of experience.

8.3.5 Discussion of the *workplace mathematics* in the task in hand.

I now consider the overall position. In the space of ten minutes Tom constructs and supports with evidence an argument that Kookaburra might have to write off approximately 25% of its debt. In the process he discards his expectation of a 50% write off. Cash flow analysis forms the framework for Tom's argument. Each step in the argument follows logical from the previous step/s and is supported throughout by known facts and circumstances particular to the case. The form of the argument is as follows:

1 Establish long term debt
2 Debt x rate of interest = annual interest burden
3 Turnover necessary to support operations has to be greater than £700 million, i.e., interest plus expenses
4 Turnover = £581m
5 Therefore cannot service debt – i.e., interest and debt repayments
6 Annual cash flow calculated
7 It is approximately equal to the interest burden
8 Therefore can only support interest
9 Therefore need to write off some debt
10 Set up equation to calculate debt which can be serviced by [first year] cash flow of £350m
11 Test 50% write off in equation.
12 £150 million free cash flow remains
13 Therefore 50% is a substantial overestimate of write off
14 25% may be better estimate
15 If so, leaves approximately 25% of cash flow for capital repayments

16 Verify by checking that if 25% of cash flow used for capital repayments then remaining principal can be repaid
17 25% is better estimate
18 So market prices not so far away

To create the argument Tom realises a cash flow analysis. As Tom constructs the argument he constructs and uses seven mathematical models – see table 1. The models are verified as they are constructed as is shown in the exhibit below.

Table 1 – Mathematical models used by Tom as he estimates the potential Kookaburra write off

Model – identified by purpose	Nature of verification/source information
1 Estimate of long term interest bearing debt	From published results
2 Estimate of annual interest burden	**To** published results
3 Estimate of turnover necessary to support debt	From published results
4 Estimate of annual cash flow	From published results
5 Model of annual debt service including capital repayments – the equation	Knowledge of term of Kookaburra's concession, suitable rates of interest and the results derived from 1 and 4
6 Evaluate impact of 50% write off by testing appropriate numbers in equation. **Answer confounds expectations.**	
7 **Expectations reset at 25%.** Test 25% hypothesis to see if capital can be repaid.	The results of 1 and 4, knowledge of term of concession and deduction that 25% write off will leave one quarter of cash flow free for capital repayments

When one looks at what Tom did, much is achieved with deceptively simple calculations. To call what Tom does calculation/modelling is a misnomer; he constructs a logical argument which disproves his expectation of a potential 50% write off and justifies a potential 25% write off, which indicates that the market prices are a reasonable guide to the current value of the debt. As Tom thinks and extemporises, the development of his argument is initially governed by his expectation of a 50% write off and subsequently by a 25% write off. The argument is firmly anchored by numbers from the accounts and their commercial meanings. Models, both mental and realised (in communication), are brought into play extempore one after another in response to facts and findings leading inexorably to verification of the revised estimate of the write off. The models are not created (i.e., composed and produced) and then used; creation and usage are contemporaneous with the exception of the equation modelling sustainable debt. The models are nested within an overarching model framework that of cash flow analysis. The numbers used in calculation are almost without exception rounded to enable ease of use. Commercial meanings are attached to many of the numbers. As Tom works, the numbers used and produced by models are derived from or verified against numbers in Kookaburra's interim results or measured against commercial facts Tom accepts as given. Tom records little; he writes one algebraic equation in his note book and punches some calculations into a calculator. Only at one point do I record Tom as pausing to think or to reflect; this occurs when his analysis does not produce the answer he is expecting. There were momentary pauses as Tom looks for more information, evaluates interim results of his work or decides upon and creates new models but essentially the argument was developed in an uninterrupted flow. This is a virtuoso performance. Or to use more conventional language it was that of an expert.

Tom's analysis of Kookaburra's situation is typical of an expert in that it, as is appropriate in the circumstances of his review, focuses on the big picture and integrates seamlessly particular facts and circumstances relating to Kookaburra and a series of simple models within an overarching cash flow framework to produce a logical verified argument. The evidence for this is mainly contained within the episode itself, e.g., the assurance with which Tom accomplishes each step, his responsiveness to the facts and the results he extracts and produces – particularly the way the results from his modelling influence and structure next steps, and the apparently effortless way he constructs a logic argument supported by evidence which he assembles as he generates the argument. Tom's performance is underpinned by an assured use of *workplace mathematics*, and accountancy and auditing practices relevant to auditing bank loan loss provisions. These skills would have been developed and honed through substantial practice.

8.4 The cash flow for Possum Networks – construction and use

8.4.1 Possum Networks – why Eric constructs the model

I now turn to the Possum Networks cash flow constructed by Eric, a relative novice. My description of Eric working highlights his uncertainties and his relative inexperience. I reflect upon this, the mental models (the idealised cognitive models) that he probably used as he worked and how the cash flow model was built and used.

For each loan subject to review Eric wrote a brief explanatory (as compared to analytical) commentary on the company's/group's latest results in the section of the credit review proforma headed 'Financials'. Each commentary generally had paragraphs on the profit and loss account, the balance sheet and the cash flow. Eric did not use the Bank's analysis for these commentaries but downloaded the company's/group's latest set of results from the internet and generated the analysis himself.

When Eric comes to the 'Financials' section of the Possum credit review, he downloads the 2002 Possum Networks accounts, reads them, and compares them with the Bank's file note. He tells me that the accounts and the Bank's review are reasonably comparable. A little later he comments that the accounts do not contain a cash flow statement because it is a subsidiary; cash flow statements are only required in group accounts. He drafts his commentary on the profit and loss account and the balance sheet. Eric then observes to me that the company has zero net cash flow [for the year] as the surplus cash has been dividended out to its holding company. He flicks through the accounts thinking. He decides to construct a cash flow for the company.

8.4.2 Eric creates and uses the cash flow model

Eric constructs the cash flow using information from the Possum Networks accounts. As Eric works, he talks to me about what he is doing. This was one of two significant episodes analysed where I was drawn into a participant's work. I think that Eric drew me in because he wanted some support. My description brings out his inexperience and his uncertainties. I leave my text to tell the story. I note where Eric sought my opinions.

Eric's construction of the Possum cash flow, a mathematical model, is divided into four distinct phases; the determination of the operating cash flow for the year, the use of the cash generated (two attempts), the determination of the year end cash, and consideration of the results.

Exhibit 8 – Extract from incomplete Possum credit review – 13/11 18.10pm version

Financials

………

An [*sic*] rough calculation of the operating cash flows shows the following for the year ended 31 December 2002:

1		**£'000**
2	Operating profit	151,271
3	add back: depreciation	38,301
4	Movement in trade creditors	(9)
5	Movement in trade debtors	(18,428)
6	Fixed asset sales*	(293)
7	Operating cash flows	**170,842**

With net fixed asset purchases of £101m, assuming interest payable equals interest paid and tax payable equals tax paid (deferred tax = n/s), these total £34m and £22m respectively. This leaves a balance of £14m.

Footnotes:

* Net profit on sales

Column in red added to aid discussion

Eric sets up an Excel table in the Possum Word template and inserts the information working down the table systematically to arrive at the operating cash flow. All the figures were extracted from the Possum Network accounts. The total was calculated using the autosum function. Row 6 was inserted after the total was set up. The trade debtors had risen during the year by £18.428m; cash from year end trade debtors is not received until after the year end, so any increase between year ends reduces the cash generated by the business in that year. Initially Eric adds the increase in the table. He hesitates clearly inviting a comment from me. I say "Did the debtors go down?" Eric thinks, changes the sign in the calculation by inserting brackets around the number, and says "It is right. Yes", affirming a rise in debtors. As Eric composes the paragraph below the table using figures extracted from the accounts, he sets out how the cash was used and calculates the unused cash to be £14 million (171 - 101 - 34 - 22 = 14). Eric pauses and saves the template.

We discuss. Neither of us is happy with unused cash of £14 million because it does not leave enough cash to cover the £30 million dividend that was paid; it should have been £16 million more. We agree the net cash flow should be almost zero, after taking account of the dividend.

Eric decides to extent his cash flow model and show how the cash is used. He extends the Excel table and continues to work down it systematically.

Exhibit 9 – 13//11 18.31pm version

Financials

…………

An rough calculation of the operating cash flows shows the following for the year ended 31 December 2002:

1		**£'000**
2	Operating profit	151,271
3	add back: depreciation	38,301
4	Movement in trade creditors	(9)
5	Movement in trade debtors	(18,428)
6	Fixed asset sales	(293)
7	**Operating cash flows**	**170,842**
8		
9	Interest paid (assume = payable)	(34,214)
10	Taxation paid (assume = payable)	(22,064)
11	Fixed assets purchases	(100,806)
12	Fixed asset disposals	5,529
13		**19,287**
14	Financing	11,754
15	Dividend paid	(30,000)
16	**Cash movement at year end**	**1,041**

With operating cash flows of around £170m the company should be able to repay it's debt within 3* years (based on the £521m level of debt (from credit app), ie includes the £149m guarantee to ME plc)

Footnote:
* Changed to 4.5 when Eric decides to use cash flow post tax and interest (170 – 34 – 22).

Again all the numbers are taken from the Possum Network accounts. The numbers in rows **9 to 12** document more accurately the information in the footnote to the original table. As the subtotal of £19 million is insufficient to fund the dividend of £30 million, Eric realises that he needs to add financing generated by the increase in non trade creditors. He types the calculation of the increase (which comprises five figures) in the formula cell of column 2 line **14.** There is now enough cash to fund the dividend. When Eric completes row **16** the model produces a figure close to the company's closing cash balance. At this point Eric regards the model as finished. He does not check his work, possibly because of his relief at getting the 'right' answer.

Eric types a new note at the bottom of the cash flow. As he does so, he refers to the Bank's review note to get an estimate of Possum Networks' indebtedness to third parties and estimates on his calculator that it will take 3 years to repay (521/171 approx. = 3). After this second attempt at interpretation, we discuss the implications of the cash flow further. Eric is not sure about his analysis and wants some guidance. I offer one or two comments but try not to get too deeply drawn in. As a result of our conversation, he alters the calculation of the payback period using the operating cash flow after deducting interest and tax instead of the pre tax and interest figure. The payback period changes from 3 to 4.5 years.

8.4.3 Discussion

a) Eric's inexperience – a relative novice at work

Eric was a relative novice when it came to constructing cash flow models for use. This is evidenced by his hope that the analysis at the foot of the table in exhibit 8, which ignored the change in financing, would be sufficient; the fact that he actively sought comments from me at various points where he was uncertain; and his cursory hesitant analysis of the results (cf. Tom's approach to Kookaburra cash flow). Eric drew me into being a contributing member of the team. This was not inappropriate as team members frequently sought informal help from each other, particularly when they were both working on the same task.

Nevertheless he constructed the cash flow successfully without looking at other examples. Eric was very familiar with the purpose and structure of cash flow statements. During the 2003 loan loss review alone, Eric would have looked at 30 to 40 cash flow statements and he had learnt how to construct them during his accounting training. His understanding and experience enabled him to approach the task methodically. His work was divided into four distinct phases: the determination of the operating cash flow for the year, the use of the cash generated (two attempts), the determination of the year end cash, and consideration of the results presented in the final cash flow. There was a clear separation between the modelling building and the analysis (cf. Tom in section 8.3). He was successful with his model building but less so with his analysis. The latter I believe was due to his lack of expertise; he did not know what to do.

b) Workplace mathematics – Eric constructs the Possum cash flow, a mathematical model

The cash flow model as realised by Eric is a mathematical model; it calculates the cash generated in the business in the year, deducts/adds non operating expenses and receipts to calculate the increase in the year end cash. The structure of the model was determined primarily by Eric while the content came direct for the Possum Network accounts, as Eric modelled particular facts and circumstances. I do not analyse Eric's detailed calculation practices here as they are similar to those described in chapter 7.

As Eric constructed the cash flow he did not refer to an exemplar. There were no guidelines in the Possum Network accounts. Its structure, which is primarily mathematical, was thus heavily dependent upon his own knowledge and skills. Therefore it is necessary to speculate about his thought processes. What he did provides some clues as to what he thought, but these were not sufficient. I use what he did and possible idealised cognitive models (ICMs) to do this. ICMs are the theoretical constructs that I have chosen to use to elaborate upon the mental models used by participants.

Eric executed the task in hand in a series of episodes. The execution of the phases follow the order of many statutory cash flow statements but only, quite appropriately, included common major adjustments. Eric's systematic approach showed he had a good understanding of the structure of the relevant cash flow model and what it is meant to show. It also suggests that he was able to use a series of personal ICMs to facilitate his execution of the task. I believe that, as he worked, he used four main ICMs, **each of which is itself a mathematical model**:

- first he calculated the operating cash flow for the year – ICM 1 underpinned the execution of this task, namely construction of table in exhibit 8;
- he then set out how the business used the cash – ICM 2 was used twice, once to underpin the explanation/calculation of how cash was used in the note to the table in exhibit 8 and again in lines 9 to 13 of model in exhibit 9.

When Eric realised that the analysis using ICM 1 and ICM 2 did not provide the desired answer, he activated two more;

- ICM 3, a methodology for calculating the change in financing arising from changes to non-trade creditors funding the business; and
- ICM 4, a model to enable account to be taken of payments to shareholders.

I believe that logic formed part of each ICM because for each adjustment Eric had to decide whether to add or deduct from the operating profit. Thus *workplace mathematics* including logical reasoning was critical to the execution of each phase. Nested within these ICMs are categories of the mind and accounting, (e.g., fixed assets, depreciation and accounts classification of debtors/ creditors) and other ICMs (e.g., various measures of profit, effect of change in debtor balances), part but not all of which were activated.

I now consider ICM 1 further to highlight its mathematical properties. It enabled Eric to estimate cash generated from the business operations by creating a framework through which he could adjust profit for period for the common significant non-cash items included those deducted in arriving at the profit for the year (depreciation and profit/loss on sale of fixed assets) and changes to cash resulting from working capital movements (debtors, creditors and stock). Eric would not have needed to activate the whole model as Possum Networks did not carry any stock. ICM 1 is a mathematical model in that non-cash items deducted/credited in arriving at profit are added back/subtracted from the profit for the year and working capital movements are added or subtracted depending upon whether they generate or absorb cash.

I also believe that Eric used further ICMs (namely 5 and 6), which were nested within the operating cash flow ICM, to work out the impact of the change in opening and closing debtors and creditors on the cash flow. Initially Eric added the increase in trade debtors to the operating profit. He hesitated immediately and sought to draw me in. I responded by saying "Did debtors go down?" – I could not see the numbers in the accounts. Eric paused, thought and then put brackets around the debtors figure in the table, that is, he deducted the increase in debtors from the operating profit saying, "It is right. Yes" with emphasis on the "Yes". He had satisfied himself, quite correctly, that a rise in debtors in a period reduces the cash generated in that period. He did not explain his thinking to me but he showed by what he said that he was satisfied that he had got it right. I believe that he used an ICM to reason that a rise in debtors reduces cash receipts in the period and hence the cash profit for the period; to be effective such an ICM needs to incorporate logical schema that enable reasoning in the event of either a rise or fall in debtors (or creditors).

I can only speculate about the main ICMs Eric used to underpin the structure of the cash flow model as I did not ask him to elaborate on what he thought as he worked. However I believe that he left sufficient clues in what he said, did and wrote to enable me to provide an outline of the mental models he used. He, however, also needed to bring a vast array of other knowledge and skills into play to create the cash flow, for example: he used IT to create an Excel spreadsheet; he read and extracted the relevant numbers and information from the accounts; he had understandings of what the accounts categories (e.g., debtors, financing costs) were; he used many of the relevant accounting concepts and techniques; and **he reasoned and set up many calculations proficiently**. Many of these skills were used effortlessly (or to be more exact with relatively little effort) and are likely to be underpinned by their own ICMs To use Wittgenstein's reasoning the knowledge and skills were part of the background and technique necessary for competent performance, i.e., the creation of the cash flow.

c) Other significant features of the Possum cash flow model

The need to create the cash flow model arose out of a task in hand – the review of the 2002 financial results. The accounts did not contain a cash flow statement. Eric considered comments on a company's cash flow to be an essential element of credit reviews. He therefore decided to create one.

Eric modelled a particular case. He modelled the 2002 results directly. Eric did not create a model and then populate with the figures under consideration. The model which Eric used belong to a family of cash flow models that are well theorized within the practice of accounting and finance. It broadly followed the programme but not the format of statutory cash flows.

All lines in the model have specific commercial meanings – all bar one being labelled. The commercial meanings of all inputs to the model were retained as the mathematical relationships were constructed line by line. Eric retained the commercial meanings within the model not only to aid understanding and interpretation, but also to facilitate the construction. Like Nunes et al's participants (1993), my participants (Eric, Tom and Sasha) in keeping commercial meanings closely attached to mathematical relationships as they worked, satisfied their need to preserve commercial meanings as they both constructed models for use and use them.

Eric's cognitive effort and work was firmly situated in the task at hand. This situatedness includes:

- the decision to build the model;
- the use of the results on which he wished to comment; and
- the fact that the meanings extracted and created are kept to the foreground as he worked.

However to accomplish the task Eric had to use his own theoretical knowledge of the purpose, construction and use of cash flow models and other accounting concepts and techniques. Both aspects – the use of the particular and theoretical knowledge – are inextricably intertwined; the particular is modelled using Eric's theoretical knowledge.

Finally Eric's initial use of the cash flow model to determine whether or not Possum Networks would be able to repay its debt from future cash flows was naïve. The interpretation involved the creation of another mathematical model, namely total debt divided by annual operating cash flow to provide an estimate of a payback period in years. His first interpretation ignored prior claims on the cash such as interest, tax and the purchase of fixed assets necessary for the maintenance of the business and any justification for using the 2002 cash flows as representative of future cash flows. This suggests that he had little practical experience of interpreting cash flows; other evidence gather during the course of the observations shows this to be the case.

8.4.4 How the cash flow was used and changed during the audit review process

Eric's Possum credit update paper and working papers file goes through the same review process as Kookaburra. Possum Inc, Possum Network's US parent, was at the time of the review in serious financial difficulty. As the Possum exposure is new to both Cliff and Tom, Eric initially spends some time briefing them as to the UK group and its business and the nature of the Bank's exposure to the group. Eric explains that he has only looked at the recoverability of the loans to Possum Networks because the Bank's loans to other group companies are insignificant. Cliff and Tom agree with this approach. In addition to reviewing Eric's assessment of the cash flow position, they both ask Eric to check that Possum Networks had not guaranteed any of the group's liabilities, other than the Platypus Electricity Bond. In this section I only deal with the loans to Possum Networks and the cash flow model.

Exhibit 10 – Extract from transcript of meeting between Cliff and Eric held after Cliff has reviewed Eric's files

C *(Looking at cash flow in credit update paper.)* .... We looked at ....the December cash flows and then made a few comments .....about how we are going to be repaid ....Well some of these ..need ..to be rethought ......the 4 and half year one **(B)**
(Short discussion about why cash flow constructed and lack of half year figures)
C ..If you ..look at the half year results versus ..the December ones for the group
E Yup
C the group ...has done significantly worse
E Yup
C So, with that in mind, it is quite bullish to say what may not be true.... What we need to do ...is ..to convert them into ratios ....because you will never be able to say the debt will be repaid [but] it is fair to [use] operating cash flows divided by ...the net debt *(sic; means net debt/cash flow)*
E OK
C .....then ...talk about that as opposed to what can be paid
E OK
C ...I am happy with what we are doing.... **(A)**
E Yeah yeah
....
C ....Can [you] look to see if they have any fixed asset commitments ... **(C)**
E Yup
C Because they seem to have huge amounts of fixed asset purchases ....Just [to]understand ...if they [do] consistently.....All I am saying is that if you are consistently spending that amount of money that number *(the ratio)* will be a lot higher because you have actually got to subtract some of that *(cost of new assets)*from that *(cash generated)...*
C And then ....we need to comment....[on] the dividends. **(D)**

E Yup……They seem to…..sweep up all the cash and dividend it ..
C …That obviously does mean …they will never be accumulat~~ive~~[e cash]
….
E Yup
C [As] the top company is single B, the chances are it will want as much cash as possible
C OK
……
C Um……. [Can] you get a feel for ..the [size of the] intercompany ..**(E)**
E In terms of whether it is third party or
C ..If there is any intercompany borrowing or lending and if a big chunk of the interest is
E Um …they……disclose that…
C Because it might ..be useful to understand ….just how much it is
E Yup
Pause
C I think that that's it [on the cash flow].

During his preliminary review of the Possum file, Cliff reviewed the cash flow produced by Eric critically, as can be seen from his comments to Eric. He affirms Eric's general approach **(A)**, but suggests Eric uses ratios instead of a payback period in his conclusion **(B)** and asks Eric to find out more about three matters **(C) to (E)**. In all cases Cliff gives reasons for his suggestions. All Cliff's points are aimed at improving the quality of the assessment that can be drawn from Eric's cash flow. Cliff does not express an opinion at the this stage.

Later that morning, Eric amends the credit update paper taking account of all Cliff's points. In particular, he changes the note relating to the company's ability to repay its loans

> "With operating cash flows of around £170m the company should be able to repay it's [*sic*] debt within a reasonable period as the debt to cash ratio post interest and tax is 4.5 (based on the £521m level of debt (from credit app), i.e. includes the £149m guarantee to PE plc). … The debt to cash ratio post fixed asset purchases is: 37.8."

Eric now evaluates position using ratios rather than payback periods.

In the afternoon, Cliff and Eric talk Tom through the Possum loan. Eric briefs Tom fully on the commercial background and explains the cash flow, bringing out, in particular, all significant points raised by Cliff. Tom confirms his understanding to Eric and Cliff by summarizing what the cash flow is showing thereby highlighting his understanding of significant numbers and issues. There is some discussion between Cliff and Tom as to the viability of Possum's business.

Eric then points out that although Possum's business is generating cash, it is also spending a lot. Tom uses this to sum up the position:

> "Well the issue is….In this particular circumstance one would expect an electricity utility to have high debt … for reasons you probably know. You would expect that [it] would be geared … up because [its] cash flows are *extremely* stable…….. So one would expect to see debt. One would expect to see it throwing off cash. Which it is. You are absolutely right that what we need to understand ….. …. the regulator's attitude to it …. Is the regulator going to require a large amount of capital investment …. this company may not be able to get [it] from its group, which may mean that this company goes bust… Then .. we are going to get our cash from [selling] the network to somebody else."

Eric intervenes and tells Tom that Kangaroo has formally offered to buy the business via the purchase of the UK group. Tom responds "Aaah.... Right... OK" The view that the Bank's loan is recoverable emerges unspoken. Tom moves onto to ensuring that Eric obtains all the evidence necessary to sustain this view.

8.4.5 What the audit team did with the model

Using the cash flow Eric was uncertain in his interpretation of the results produced by the model. He struggled to find both the appropriate ratio and its description; the commentary reviewed by Tom was Eric's fourth attempt. Eric's struggles arise from inexperience since prior to his work on the 2003 audit he had little practical work experience of creating and using cash flows; it is noteworthy that he took up all Cliff's suggestions when he amended his draft paper. Cliff's review of Eric's work is concerned with the numerical results produced by the model and the commercial background. Apart from guiding Eric how to use and express ratios, he focuses on identifying commercial constraints on Possum's use of its future cash flows. After Tom was briefed about Possum by Eric and Cliff and he had made sure that he had understood the model (by talking through his understanding with Eric and Cliff), he used the model and its results and his general knowledge to reason why the Bank might or might not require a provision against the loans to Possum. Cliff's and Tom's uses of the model are context driven. Both were experienced at using cash flow models and knew what they ought to illustrate. They used this knowledge as they work.

Innovation in practice When Cliff guided Eric to use the ratio model to express the mathematical relationship between the company's debt and free cash flows, two important events occur simultaneously. Cliff taught Eric and Cliff imported a new theoretical perspective into the Possum credit review. It, however, was driven by a context, namely, the review of Eric's working paper. Again theoretical and context considerations are inextricably intertwined.

Comparison between Possum and Kookaburra The processes by which the team obtained an understanding of and assessed the Bank's position were similar. Through discussion the team reached an agreed consensus as to what the models showed. The Possum cash flow model was, like the debt valuation model for the Kookaburra loan, pivotal to the audit team's consideration of the Possum loans; it led to the conclusion that no bad debt provision was needed because it emerged during the discussion of the model that the company was to be sold to Kangaroo. Perhaps the main distinguishing feature between it and the Kookaburra case was that consideration of the model and its results triggered primarily a discussion of relevant commercial considerations rather than an analysis of its mathematical results and their implications.

[redacted]

Original model reconstructed

Table 2.

Valuation of goodwill

		Year 1	Year 2	Year 3	Year 4	Year 5	Total
Accounts acquired in the UK ×5	9225	1,845	2,709	3,573	4,437	5,301	5,301
Plus: Additions of 864 per year for 5 years	12,960	864	864	864	864	864	864
	22,185	2,709	3,573	4,437	5,301	6,165	6,165
		47%	32%	24%	19%	16%	
Average charge per account (£20 per month) £15 per month	£180	240	240	240	240	240	240
Total revenue per year	£000s 3993	650,160	857,520	1,064,880	1,272,240	1,479,600	5,324,400
Cost of sales per year (178269 / 5 \x 12 months)	(2140)	427,846	628,203	828,560	1,028,916	1,229,273	4,142,798
Plus: Increase of COS in line with increase in sales		200,357	200,357	200,357	200,357	200,357	1,001,785
Gross Profit	1853	21,957	28,960	35,964	42,967	49,970	179,818
Gross Profit %		3%	3%	3%	3%	3%	
Other Costs:							
Administration	(111)	22,267	22,267	22,267	22,267	22,267	111,336
Depreciation	(736)	245,273	245,273	245,273	0	0	735,818
Net Profit	1006	(245,583)	(238,580)	(231,576)	20,699	27,702	(667,337)
Net Profit %		-38%	-28%	-22%	2%	2%	-13%
Goodwill balance	(880)						880,089
Net Profit after goodwill	126						(1,547,426)

← Original model reconstructed because it was deleted as soon as extended model complete and no copy of original working paper kept.

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8.5 A goodwill valuation reconstructed

8.5.1 Introduction

During phase 1, I observed Joan, an experienced in charge, develop a second version of a goodwill valuation – table 2. Joan substantially reconstructed the goodwill valuation, a mathematical model, in about 45 minutes. This proved to be inadequate as it turned out that she did not have enough information about the purchased business and its prospects at hand for the valuation to be completed successfully. In this section I analyse the reconstruction of the spreadsheet model in depth in order to reveal Joan's model building methodology. Many of the processes revealed turn out to be or to involve *workplace mathematics.* I also show how Joan and the audit manager, Simon, used this version of the model to progress the task of evaluating BFJ's goodwill depreciation policy.

8.5.2 Commercial background

On the day I observed Joan, she was part way through the fieldwork on the BFJ audit. Late in the morning Simon asked her to progress the work on the goodwill valuation as the auditors of the Germany subsidiary "were not happy with [the subsidiary's] allocated share of the goodwill."

BFJ is a UK company. Its business includes putting together soundtracks for piped music and selling them to retail companies, e.g., clothing companies. During the course of 2001, BFJ was acquired by a US corporation. Later in that year a 50% UK affiliate of the US corporation sold its business, which comprised selling piped music transmitted via satellite transponders, to BFJ and other European subsidiaries for US $3 million. Substantial goodwill, i.e., the difference between the value attributed to the net tangible assets and the purchase consideration, arose with BFJ bearing £880k and the German subsidiary £250k.

The task in hand for the audit team was to satisfy themselves that amortizing (writing off) the goodwill in the BFJ accounts over five years on a straight line basis was reasonable. The assessment was to be made by comparing the results from a valuation of the goodwill with the price paid for it, the valuation being a 'forecast' of the pre tax trading profits from the purchased business for the five years, 2002 to 2006. Joan had prepared an initial draft valuation using a global five year calculation (refer to table 2). It showed that the total pre tax profits for the five years exceeded the consideration for the goodwill and hence supported the view that the depreciation policy was reasonable. There was a mistake in this calculation; it did not take account of selling costs relating to new customers.

8.5.3 Reconstructing the goodwill valuation – grounded analysis

The goodwill valuation is a mathematical model, being a calculation of the results of predicted future trading. It is a much larger model than Eric's Possum cash flow. Its reconstruction required Joan to make more decisions and judgements about both its structure and contents. Grounded analysis of Joan at work provides additional insights into the model building process. Table 3 highlights Joan's construction processes and summarizes the *workplace mathematics* used. (Source: detailed observation notes and table 2.) The coding used – see the headings in table 3 – is a variant of coding system used in phase 1. The variant implicitly emphases thinking as well as doing, as Joan articulated some of her thoughts as she amended the spreadsheet. It also 'chunks' together and suppresses the arithmetic and spreadsheet algebra to foreground spreadsheet construction processes. This is a *post hoc* justification of the coding after successful experimentation.

Table 3 needs to be read in conjunction with table 2 and subsection 8.5.4.

Table 3 – Description and analysis of Joan's reconstruction of the goodwill valuation

Steps coded	Reviewing/using model	Reviewing working assumptions	Changes to model or other action	*Workplace mathematics*
1	Assesses original model critically			Re-understands what valuation is showing
2		Questions new customer numbers – too low? **(commercial)**	But does nothing	Compares with client expectations which are much higher
3		Decides to build up results on year by year basis in model **(model)**		
4			Adds six columns for years 1 to 5 and the totals to original model	
5	Decides to fill in numbers for year 1			
6		And to use original **model** assumptions		
7			Creates year 1 trading account in column 2 from results for first five months (already itemised in model)	Mostly annualises figures for five months, subtotals and totals
8			First draft figures for years 2 to 5 created by copying cells from year 1	
9	Looks critically at customer numbers in revised model			Checking model integrity; numbers not cumulating properly
10		Decides for time being to accept annual estimate of net new customers **(model)**		
11			But amends model to cumulate annual increase in customer numbers	Total for year 1 customers becomes repeat sales for year 2 etc
12			Totals figures for 5 yrs in total column	Using autosum function key
	No clues to Joan's thoughts about model construction that lead to next action			
13			Deducts goodwill from total estimate of 5 years trading	Uses autosum key **Thus sets up key comparison**
14	Looks at model again	Reviewing depreciation charge against cost of fixed assets **(commercial)**		Spots assets over-depreciated – cost written off in three years

Steps coded	Reviewing/using model	Reviewing working assumptions	Changes to model or other action	*Workplace mathematics*
15			So depreciation deleted from years 4 and 5	
16	Reviews results			Shows large profit
17	Looks at model and thinks			
18	**Observer prompts**	**Are all sales costed properly?**		
19		Reviews cost of sales calculation in model **(model)**		Works out cost of sales to new customers not included in trading account
20 steps 18-20 repeated			Amends model substantially to cost sales to all customers at rate of £180 a year	Amends model logic and calculations
21	Reviews model – produces large losses			Considers results
22		Considers sales per customer too low **(commercial)**		Evaluates make up of sales figures
23		Reviews revenue per customer **but not** customer numbers **(commercial)**		
24			Amends sales per customer from £15 to £20 per month	Alters calculations And justifies by reference to actual sales figures
25	Model still producing losses			Considers results Identifies that cost of sales may be problem
26		Reviews royalty position by referring to contract. Costs capped but can't find how **(commercial)**		Capping may reduce costs considerably
27			Decides to talk to manager	
28			Deletes original global calculation and prints spreadsheet	

8.5.4 Discussion

Numbers in bold in text refer to the coding steps.

a) The task performed

Joan's task was to re-look at the first draft valuation of the goodwill and take it further. The original draft, reconstructed in table 2, was set up on an Excel spreadsheet. It showed that the projected total pre tax profits for the five years (subsequent to purchase) exceeded the consideration paid for the goodwill. Joan decided (**1 to 3**) that she needed a five year model showing a build up of profits on a year by year basis. Her starting point was the global five year calculation and the annualisation of the results for last five months of 2001, i.e., the five months subsequent to the purchase. She reconstructed the original model through the iterative process illustrated above – it took about 45 minutes (**4 to 25**). Part way through the reconstruction, Joan realised (**18 to 19**) that the spreadsheet contained a modelling error after I had noticed it and prompted her – the direct cost of sales figures were not increased in line with the increases in sales in years 2 to 5. Joan corrected the error. Once Joan had a working model, she realised (**25 to 28**) that she did not have enough information about the purchased business to complete it satisfactorily, so she went to see Simon, her manager, to discuss how to progress things further.

I decided to prompt Joan (**18**) as it seemed natural to do so – we were both looking intently at the model. My motivation was partly determined by the fact that I wished to see Joan progress the model as far as she could while I observed. From what Joan said to Simon subsequently, she realised there was error in the original calculation – see below.

b) Common features

There are a number of features common to the way in which Joan and Eric constructed models. The need to construct the goodwill model was driven by the task in hand – consideration of the reasonableness of a depreciation policy. The model used was a common and well theorized valuation model for calculating/evaluating goodwill. Both were Excel spreadsheet models with Excel performing all calculations. The facts and assumptions relating to the particular case under consideration were modelled directly. Furthermore as the reconstruction progressed commercial meanings were, generally, kept attached to the numbers in the model. However aspects of the building process were different.

c) Model building

Joan's starting point was the extant model. Consequently the structure of the revised model was derived substantially from the extant Excel spreadsheet and the Excel programme functions substantially determined the building processes. Joan's model building process involved five inter-related steps: considering the methodology for justifying the goodwill figure; thinking about the meanings generated by the spreadsheet model in construction; considering the model/commercial assumptions upon which the calculations are based; amending the computer model/spreadsheet; and using the spreadsheet model to draw conclusions. On the day I observed Joan, she did not consider the methodology – she did this while constructing the original and final versions. The reconstruction of the spreadsheet was highly iterative. Joan went round a loop eight times (starting points of each loop – **1,5,9,13,14,16,21 and 25**); the loop comprised assessing spreadsheet model/results, reviewing the efficacy of a model or commercial assumption and, where appropriate, amending the spreadsheet. Broadly in the early stages of the reconstruction, review of the spreadsheet model logic and model assumptions predominated while in the later stages emphasis was on commercial assumptions and the results produced by the model. The process was a holistic process in contrast with the traditional theoretical paradigm (discussed in subsection 6.4.2) which emphases the distinction between the 'real world' situation modelled, the model and interpretation of the model results.

Joan's approach was predominately heuristic. After she decided to reconstruct the model showing the projected trading results for each of the five years, she created the framework (steps **5 to 8** and **12**) for the model by generating year 1's results in the column next to the original calculation such that both had an identical format and then copying year 1 results into the columns for years 2 to 5. 5 year totals for each category in the trading account were created in column 7 of the model. At this stage neither the numbers nor the model logic were correct. Joan then set about making the model work. She worked systematically down the rows altering either the model logic (3 amendments) or the commercial assumptions (3 amendments). She made five substantive changes. She altered the model logic to cumulate net annual increase in customer numbers (**9 to 11**); she set up the key comparison by deducting the amount paid for the goodwill from the total net profits (**13**); she corrected depreciation charge (**14 to 16**); she remodelled cost of sales to correct the failure to take account of the cost of making additional sales in the original model (**18 to 20 twice** – involving changes to both model and commercial assumptions); and she revised the estimated of sales per customer (**22 to 24**).

d) Aspects of workplace mathematics usage

– **Repairing model logic**. Two examples are discussed below.

(**9 to 11**) Initially the sales figures did not cumulate properly; the annual net growth in the one year's sales was not included in the repeat sales for the following year.

Table 4 – Extract from goodwill valuation; <u>adjusted</u> customer figures for years 1 and 2

	Row*	Year 1	Year 2
Accounts acquired in the UK [Repeat sales]	A	1845	2709
Plus additions of 864 per year for 5 years*	<u>B</u>	<u>864</u>	<u>864</u>
	C	2709	3573
***Lettering observer's annotation**	D		
*Net new sales	E	47%	32%

Joan used the Excel copy function to adjust the model. She copied the total customer figures for year 1 into the year 2 cell for repeat sales. The Excel programme adjusted the repeat sales figures for years 3 to 5 automatically.

(**18 to 20**) Joan took two attempts to get the cost of sales figure to be logically consistent. Her approach was experimental and idiosyncratic. The starting framework had identical sales costs for years 1 to 5. First of all Joan instructed the model to calculate the annual percentage increase in sales for each year; she calculated the result for year 1 (cell E1) using spreadsheet algebra, being cell B1/ cell C1 * 100 (rather than 864/2709 *100) and copied the result to corresponding cells for years 2 to 5. She then instructed the model to calculate the cost of net new sales in **row G** by applying the annual percentage increase to the annual cost of sales figure (the latter figure being at this stage the same for each year). The alteration did not work. After a second prompt from the observer Joan noticed it led to a decreasing cost for new sales because of the reducing percentage increase (see **row E** above) and so she took a different approach.

Table 5 – Extract from goodwill valuation; <u>adjusted</u> cost of sales figures for years 1 and 2

	Row	Year 1	Year 2
Cost of [repeat] sales per year (178269/5 x 12 months)	F	427,846	628,203
Plus: increase of COS in line with increase in sales	G	200,357[1]	200,357

She realised that she needed to cumulate the costs for repeat sales (as they were originally set as being the same each year) and that the cost of net new sales was the same for each year. So using the Excel autosum function she put the total of the costs of sales for year 1 into the repeat cost of sales cell for year 2 and set cell G2 for year 2 to 200,357. She then copied year 2 into cells for years 3 to 5. The mathematical logic of the goodwill valuation itself, the model assumptions and the numbers were all now consistent. The spreadsheet had been made consistent by changing one number and using a combination of the Excel functions. Joan's approach here can be described as experimental.

– **Mathematical models used.** The goodwill valuation, which is itself a mathematical model, incorporates and uses other mathematical models. In the background is the trading account. Nested within the model itself are a number of other smaller mathematical models; year 1 results were generated by annualising the results achieved by BFJ in the first five months of its ownership (straight line extrapolation used); tangible fixed assets purchased were depreciated (written off on a straight line basis over time); an arithmetic progression (not recognised as such) was used to generate sales growth; and the sales income was computed using an estimate of revenue to be generated annually from each customer. These models were part of the bedrock practices[2] that Joan brought into play as she reconstructed the goodwill valuation. She did not reflect upon their validity but used them as a matter of course.

– **The modelling process.** Joan's approach to the remodelling was mainly trial and error, rather than analytical. The evolving extant spreadsheet model guided Joan as she changed model assumptions and corrected errors. Unlike the other models discussed in this chapter, Joan worked specifically at getting the key parts of the model to work mathematically. It is worth noting that Joan only sought to make the key parts of the model work – the customer numbers over the five years were never totalled properly and the ultimately unused percentage increases in customer numbers in years 2 to 5 were not deleted from the model.

---

[1] Assumption (unspoken and, I think, unrecognised) that in year 1 all additional sales would be made in last seven months of year. This has the effect of increasing the unit cost of sales and the losses. Joan starts addressing the excessive cost of sales almost immediately – see **25 to 27** table 3.

[2] Following Wittgenstein, these are practices that are followed as a matter of course and without reflection as they have become second nature.

e) Model usage

By the time Joan got to step 24 she considered she had created a working model. She interpreted the model mathematically; it was producing a large deficit, not a surplus as expected. She thought about the results and decided that the cost of sales figure looked wrong. She referred to the royalty contract and found that the annual royalty cost (i.e., a cost of sale) was capped but could not see how. Also at an earlier stage she had decided not to use client expectations of future sales growth. She therefore decided to talk immediately to her manager, Simon, as she realised she did not have enough information to complete the valuation satisfactorily.

Exhibit 12 – Extract from observation notes; Joan discusses the position with her manager, Simon.

1.10pm J goes to see Simon with spreadsheet and files
Shows Simon copy of spreadsheet
J "I have spotted mistake in the original goodwill calculation."
S looking at sheet says "Not good news."
J "......net loss......"
S thoughtfully "....must have done something wrong."
J ".......there is a cap on transponder costs in the contract......"
S " We need better estimates of costs from K, *[managing director of BFJ]*,.......re sales and costs.........need to talk to K or D........needs thought."
J and S look at files and purchase contract
J points out where limit is placed on transponder costs
S [and J] "Where did you get cost of sales?" "From P & L a/c" "In this file?" "Yep. Transponder costs £116k and royalties £40k." J shows S
S "We need to come up with some more accurate costs."
J "Should I do today or do I need to go and see them."
S "We need to use their projections......Then test sensitivities.....Too many uncertainties in our calculations." *[Implicit that J will go to client]*
J "Let me know if I need to phone K this afternoon."

1.25pm J returns to her desk.

The interim results produced by the goodwill valuation, i.e., through calculation, provided a focus for the discussion (lines 1-10). They enabled Joan to acknowledge the error in the first draft calculation (lines 3-5). Simon looked at the results produced by the model and appreciated they were problematic (lines 4-6). While looking at the spreadsheet and contract, Joan shared with Simon part of her understanding of possible problems with the model assumptions (lines 8-14) and Simon guided Joan on how they should take the issue forward (lines 15-20). He told her that they needed to use the client's projections (for sales and costs) as there are "too many uncertainties in our calculations". A way forward was agreed.

All the work observed and discussed was part of the practice of auditing. *Workplace mathematics* was central to this auditing; it was, as is illustrated above, essential to understanding the original goodwill calculation, the reconstruction and extension of the model and understanding the new results.

It was also essential to its ultimate use, namely showing that the forecast pre tax trading profits for five years were greater than the price paid for goodwill. Whether the use of the valuation as documentary evidence for the depreciation policy adopted by the company is *workplace mathematics* is arguable. It is certainly part of the practice of auditing and on balance I think that practice incorporates *workplace mathematics* as extracting meaning from the evidence requires a reader to engage in *workplace mathematics.* These assertions bring me to consideration of the relationship between the tasks in hand observed, the creation and use of mathematical models, and work practices and *workplace mathematics.*

8.6 Concluding remarks; modelling and *workplace mathematics*

8.6.1 Modelling

How models were used

This chapter deals with four significant mathematical models used by my participants. In all cases the results were used to reason about situations or possible situations in the 'real' world. The debt valuation model was used to assess the adequacy of the Kookaburra bad debt provision. The Kookaburra cash flow was used to predict how much debt Kookaburra might have to write off if it were to be reconstructed. The Possum cash flow model showed how the cash generated by the business in 2002 was used. It was also used to speculate about how long it might take to repay the company's debts. Joan's valuation of the goodwill of the recently purchased business was used to assess (not observed) the reasonableness of the company's depreciation policy for purchased goodwill, i.e., the price paid for it. These uses are within those envisaged by the traditional modelling paradigm (see section 6.4). In all cases both modelling and usage incorporated *workplace mathematics.*

The models, however, served other purposes:

- Cash flow analysis provided the framework for both realised cash flows and the logical argument constructed by Tom.
- The Kookaburra bad debt provision was increased so that the carrying value of the debts at the year end was their market value; the model was partially realised by action in the 'real' world, an example of *workplace mathematics* realised in action.
- Other than the Kookaburra cash flow, all the models were used to focus discussions and as audit evidence. Although these uses themselves are not *workplace mathematics,* participants used *workplace mathematics* as they discussed the models or looked at the evidence.
- In the last chapter, models were used to order, simplify and display information or a position and to make and present various forms of comparisons. This usage is distinguished from using models to reason with as the prime purpose is to present/display/summarize information.

Distinctive features of usage observed

- Prior to use, time was spent understanding the mathematical structure and contents of the model and the results presented.
- In meetings time was spent reaching an agreed consensus as to what the model showed.

- The picture presented by the model or the results were used to reason with, including when the model was in the course of construction, e.g., reassessment of the Kookaburra provision by Eric in his meeting with Cliff, emergence of view that Kookaburra is underprovided, and appreciation of need to amend the goodwill model to cumulate sales and adjust cost of sales.
- Participants from their knowledge of the commercial background had expectations of what the model would show. This affected their usage, e.g., when Tom set up the algebraic equation to model the repayment of the Kookaburra debt, he did not solve the equation but used it to test his hypothesis of a 50% write off; and when the reconstructed goodwill model produced a large deficit, Joan assumed, quite rightly, that she did not have all the information she needed for her model.
- When models were used or discussed, other mathematical models were referred to or formed part of the background.

All these aspects of use incorporated *workplace mathematics* – discussed in detail in this chapter.

Distinctive features of models and model building

- Models were built only if no satisfactory model was in existence and readily available.
- The need for a model was driven by the task in hand.
- The particular facts and circumstances under consideration were modelled. On the other hand the models used were variants of common well theorized accounting models.
- Models built/adapted placed emphasis on relationships considered to be key.
- Commercial meanings of all significant numbers within models were kept attached to those numbers as situations/events were modelled. The meanings enabled participants to evaluate the effectiveness of the model in construction, as well as facilitating use.
- Many smaller mathematical models were nested within the more substantial models.
- Participants' assessed results for reasonableness enabling error repair and other adjustments.

The minutae of making models work / model repair

- Model building **observed** fell into two categories. Where an extant model existed, that model substantially structured the form of the new model generated. Where a decision was made to construct a new model, the structure of the model was determined by the participant's extant knowledge and skills.
- Building processes, including creating the mathematical structures, were empirical and iterative and were based upon the facts and circumstances under consideration.
- It is easy to overlook the contribution of the analytical. All significant models used were based upon well theorized models of accounting.
- Many other skills, many of which had become bedrock practices, were brought into play as the models were constructed/used, e.g., such as arithmetic, simpler

commercial models, spreadsheet construction techniques and the ability to extract commercial information from accounts and other documents.

- Participants had a phased/iterative approach to building. My analysis of Eric's construction of the Possum cash flow showed that there were four phases to Eric's construction, each having a mathematical framework which was underpinned by a commercial objective. Joan's reconstruction of the goodwill valuation was iterative; at the beginning of each iteration, Joan assessed an aspect of the model and then amended part of the model, if appropriate. Tom's Kookaburra cash flow model can divided into three phases.
- What is distinctive about Tom's modelling was that he that constructed and verified a sustained logical argument seamlessly; in particular he responded to interim results, using them to take forward the argument even where they were unexpected.
- This leads to a very important observation. Participants did not move between considering the situation in the 'real' world and the model in the way envisaged by the traditional paradigm. By modelling particular facts and circumstances and attaching commercial meanings to the numbers modelled, they kept both situations to the fore as they modelled situations and used the models.
- Calculations in spreadsheets were all performed electronically, with the input being derived from facts and circumstances under consideration. The practices were comparable to those described in the previous chapter.
- **Error repair** In section 8.5, we see Joan repair logic errors, a classic form of error. Other types of 'errors' arose, e.g., Eric hoped that modelling only the Possum operating cash flows would be sufficient and Eric's and Cliff's assumption that Bank's bad debt provision for the Kookaburra loans was likely to be right. Significant 'errors' were discovered and opinions reassessed in the ordinary course of carrying out work where team members, either individually or collectively, were committed to the meaning and implications of the results produced by models.

Much was or involved *workplace mathematics.*

Models as part of auditing

Listing key findings fragments the work practices observed and fails to convey the teams' competence, including the performance of *workplace mathematics* which was embedded in the practice of accounting and auditing. In this chapter we have seen a relative novice, an experienced in charge and an expert construct and use mathematical models. Eric constructed a model successfully using his knowledge carefully and systematically. He barely used the results produced and did so very hesitantly because of his relative inexperience. Joan created a goodwill valuation that worked. She used the results to conclude that she needed more information about the purchased business in order to create a satisfactory valuation. Tom's work was that of an expert. As he constructed and used the model, he constructed a logical reasoned argument supported by evidence leading to a conclusion. The construction and usage was a tightly integrated performance. Eric's and Joan's models were constructed as texts while Tom's was extempore.

Models played a key role in teamwork. They both focused discussions and formed the basis of them. Teamwork was also important in both constructing and using models; work was shared between team members and they worked together collaboratively enabling shared analysis and judgements to emerge, e.g., during the review of the Kookaburra debt provision.

Mathematical modelling integrated into accounting and auditing practice was an essential part of my participants' everyday work. They not only constructed fairly substantial models but their work is redolent with numerous small models, e.g., working out changes in year end debtors/creditors, estimating annual interest on a company's debts, calculating forecast sales using particular assumptions, calculating various ratios and percentages, and setting up numerical comparisons.

8.6.2 Postscript on calculation practices – oral practices

My participants used calculations in discussions and as they articulated their thoughts. Most calculations were either taken from the texts under discussion or were created using key numbers from those texts. Figures were rounded to two three significant figures when identifying quantities and to enable easy manipulation of numbers. Rounding practices occasionally broke arithmetic rules. Calculations were often articulated before they were performed; sometimes answers were only estimated and sometimes were not even performed. Meanings, including units, were kept attached to numbers as calculations were set up and where they were executed they were usually performed mentally, though Tom did use the calculator for the calculation of the interest on his estimate of the long term debt possibly because the numbers, including the place value implications, were not obviously malleable.

In the next two chapters I will, amongst other things, discuss calculation practices used in narrative texts and the practice of using exploratory calculations.

I now turn to and analyse part of an observation that I found very unsatisfactory.

9 Quantitative information in narrative: as information, reasoning, warrant and rhetoric

9.1 Writing text: sometimes *workplace mathematics* in action?

In this chapter, I discuss the role that quantitative financial information plays in my participants' texts and argue that creating and using texts containing such information often involves *workplace mathematics,* sometimes in unexpected ways. Occasionally from a mathematical perspective the usage results in the presentation of incomplete and/or unsound arguments. This does not mean that they are invalid from a commercial perspective; arguments may not be completed for good reasons, e.g., their usage is superseded by other considerations or they are intended only to be indicative of events/possible outcomes. However this does not stop the usage being *workplace mathematics*. Initially, I formed the view that such usage was generally not *workplace mathematics* because the usage of mathematical relationships either is not explicitly evidenced in the text or is simply not there. As I puzzled over the role of the numbers and mathematical relationships in some texts, I realised that they play a major and varied role in participants' narratives and that much is *workplace mathematics*. In addition to providing support for arguments as a contribution to inductive or deductive reasoning used, they support the narrative by warranting statements and enabling rhetoric. This takes one a long way from school and undergraduate mathematical practice! But it reflects what happens in commerce.

My participants produced many texts as they worked. Some comprised carefully crafted narratives, e.g., the Audit Committee Paper. Others were working papers providing evidence to support audit opinions. Generally these are notes not polished papers; as they evolve parts are not refined or fall into disuse while other parts are refined. Some texts are essentially numerical in nature, others narrative. In this chapter I consider narrative texts. In particular, I analyse two sections of one piece of text in depth, namely Eric's first draft of his working paper on Wallaby. This narrative text was seminal to my thinking because I found it very unsatisfactory both as narrative and as a summary of the events and of the likelihood of the Bank recovering its loans. Trying to make sense of it from my analytical perspective revealed that there is an unexpected richness to what it means to do *workplace mathematics.*

Below I analyse substantial parts of two sections of Eric's working paper, namely 'Update on credit position' and 'How are the Bank going to get their money back?' I chose to analyse the composition of these texts rather than the usage of an already created text, partly because data collected in relation to text composition is more accessible to analysis than data of persons reading texts and partly for the reasons stated above.

9.2 Update on credit position

9.2.1 Introduction

This section of Eric's working paper provides a short update on Wallaby's debt position, and its current and prospective trading position (including disposals). Two paragraphs on another topic are not reproduced to preserve confidentiality.

Exhibit 1 – Extract from Eric's working paper

> **Update on credit position**
>
> A) The level of junior and senior debt has reduced as result of repayments made by Wallaby. (1) These repayments have been funded by the disposal of various investments mainly the £98m from Wombat.(4) The total level of notes outstanding at 30 September was £623m.(5) This is ahead of schedule by £100m-120m.(9) A further repayment was made in October, Wallaby are now able to purchase junior notes on the market.(10)
>
> B) .... (2)
>
> C) .... (3)
>
> D) Wallaby have set the following goals for their 2004 budget:
>
> - gross margins of £[*sic*] 27% by March '04
> - operating costs below £425m by March '04
> - positive operating cash flows for 2004 as a whole.(6)
>
> E) They are forecasting sales to increase quarter on quarter, with total sales of £1.6bn in FY2005 (y/e March).(7) Operating cash flows are expected to be positive (pre exceptionals) and overall cash generation is ahead of schedule due to disposals and improved working cap[ital] m[anagemen]t.(8)
>
> F) This improved performance should result in Wallaby being able to repay a further £109m in Junior notes by March 2004, at this point adequate cash surpluses should be held to repay the rest of the Junior notes.(11)
>
> **Footnote:** The underlinings and the letters and numbers in red are my additions.

9.2.2 Creating the text

Almost all the information in the text was obtained from the Bank's review of Wallaby's operations. Eric checked some details with the Bank relationship manager after he had written the text. The Bank review is a note written by Bank staff to support the Bank's credit decisions on Wallaby; it is a detailed but fairly superficial financial commentary on Wallaby's current and expected future position, leading to an assessment of the nature of the Bank's credit risk.

The numbers in red indicate the order in which the sentences in exhibit 1 were composed and typed. Eric worked through the Bank review systematically from beginning to end. As he read, he marked up the note with an orange highlighter, and typed some of the information he considered relevant into his template. Eric spent between 15 and 20 minutes reading and typing this extract.

Eric told me as he worked that he would review Wallaby's accounts himself so he would only read the Bank's comments relating to published results. So most of the credit update section, other than the comments on the banking relationship which are not reproduced here, is a commentary on the progress with respect to the repayment of the Junior loan notes and the trading forecasts for the years to March 2004 (and 2005).

Eric's task was to select and summarize information that not only provided an update on Wallaby's credit position but would also enable the audit team to assess Wallaby's ability to service and repay its existing debt. His discussion of the position with respect to the repayment of the Junior debt, paragraphs A and F, is a reasonably accurate summary of what is in the note. His comments on the trading position, paragraphs D and E and part paragraph F, are very selective and do not form a summary of the position as set out – possibly because he intended to review the published results himself – and his comments are arguably an inaccurate reflection in one respect. He did not pick up the comment that "market conditions remain tough" for Wallaby and concluded that Wallaby are "forecasting sales to increase quarter on quarter". By the time Eric finished compiling the first draft of the working paper, he had effectively revised his views on the trading position. In the conclusion he states "the situation remains volatile as sales decline." The comments on Wallaby's operating cash flows pre exceptional items, paragraph E, are a fair reflection of comments in the Bank review. In the conclusion box of the template, he adds "the operating cash flows post exceptionals remain negative". Both the change of view and the provision of supplementary information in the conclusion demonstrate how the template itself is work in progress.

9.2.3 Eric's comments on the Junior loan notes – paragraphs A and F

Was Eric using *workplace mathematics* when he extracted and summarized the information in paragraph A? The process that Eric used to construct his text strongly suggests the contrary. Eric's text is derived directly from comments interspersed throughout the Bank review. (Evidence: order in which review read and sentences were typed.) Eric's text sets out the position with respect to the early repayment of the Junior loan notes. Although the paragraphs contain some quantitative information and analysis, it is essentially reportage. Eric selected relevant information relating to the loan notes from the Bank review in order to give an overview of the current position (i.e., current at the date of the review). In the process he did not alter the source text much. In a couple of cases he changed meaning slightly; these changes do not relate to the numbers but occurred as Eric précised the text. Through the process Eric constructed an informative narrative about the repayment of the Junior debt; in particular he ordered his selections from the Bank's review to create a coherent narrative. It is to be noted that he did not refine and redraft the note, except for one amendment.

Superficially Eric does not appear to have used mathematical reasoning or mathematical models. On the other hand numbers and words conveying an improvement scenario (e.g., 'reduced' and 'A further repayment') do play a key role in Eric's text. So what role do all these play in the creation of the text?

Eric's text does three things in paragraph A: it sets out the history of the Junior debt repayments; it quantifies the position as of 30th September including the extent to which it is ahead of schedule; and it indicates how repayments have been achieved. Furthermore in paragraph F, Eric reports that by March 2004 (four months after Eric's review) Wallaby should be able to make a further specified repayment and should have enough cash to repay the rest of the loans. The text creates an incomplete outline of the history of the Junior debt and its repayment. The facts and opinions chosen paint a positive picture of the Junior loan notes from the Bank's perspective.

The argument in Paragraph A coheres but is not complete from a logical/ mathematical perspective. With further work, Eric could have used the Bank review to construct a complete picture of the position. He did not do so presumably because he considered his commentary to be sufficient. But he has in effect created the fragment of a model/proof as is demonstrated by my re-presentation of the information in table 1 below.

Table 1 – Current and prospective position with respect to the Junior loan notes

	£m	£m
Opening balance		X
Cash generated by sales of investments used to fund repayments [and other activities?]	**98+**	
Repayments		**(X)**
Outstanding at 30th September (£100m-120m ahead of schedule)		**623**
Further repayment in October		**(X)***
Outstanding at time of review		X
Repayment expected in March		**(109)**
Expected balance at Wallaby year end, 31st March 2004		X .
*** so now able to purchase loan notes in the market**		
Footnote: The presentation used in this table is typical of accounting practices that reconcile opening and closing balances.		

(Source: text in **red** *derived from Eric's text.)*

Eric created his narrative. He did not cut and paste it directly from the Bank's text. He constructed it by taking individual sentences from the review; some sentences were amended slightly. Eric created the narrative by the way he ordered the sentences. As can be seen from table 1, the text is essentially a fragment of a mathematical and financial proof/model created in narrative form. It is an incomplete mathematical proof/model populated with relatively few numbers. *Workplace mathematics?* Probably, as is explained below.

Exhibit 1a – Paragraph A from exhibit 1

> **Update on credit position**
>
> A) The level of junior and senior debt has reduced as result of repayments made by Wallaby. (1) These repayments have been funded by the disposal of various investments mainly the £98m from Wombat.(4) The total level of notes outstanding at 30 September was £623m.(5) This is ahead of schedule by £100m-120m.(9) A further repayment was made in October, Wallaby are now able to purchase junior notes on the market.(10)

The information in the sentences in the middle of paragraph A report significant financial numbers while the words underlined carry connotations of arithmetic operations in addition to describing events/situations. The numbers and words describe events and create the logic for the model fragment. But I suggest they do much more. The figures provide measures of the amounts involved. Some help to give authority to the narrative, e.g., "£623 million still outstanding"; others help to warrant the text, e.g., "£98m from [the sale of] Wombat". The words underlined paint a favourable picture with respect to the junior debt, e.g., 'reduce', 'repayment', 'ahead of' and 'further'. Together they create rhetorical force. Imagine what Tom might have said if there were no numbers and the text was more neutral in tone "OK, OK, OK. How big were the repayments? How much did we get for the sale of investments? How far ahead of schedule are we? How much is there still to repay?" I suggest that the numbers and the underlined words are used to strengthen and warrant the description and the argument giving them rhetorical force, in addition to supporting the logic of the narrative.

So is using numbers, operations, and descriptions in these ways part of *workplace mathematics?* The text is a narrative of the progress to repayment. It does not contain sentences recognisable as mathematical statements, like for example, "£x million loan notes have been repaid reducing the outstanding debt from £y million to £z." But Eric has constructed a text with rhetorical force (possibly comparable to the use of quantitative information and some statistics in some daily newspapers) in addition to an incomplete fragment of a proof/model including the justification of some statements made (comparable to a school geometry exercise). I think it is arguable that both the creation of the model fragment and the rhetorical usage constitute the use of *workplace mathematics.* I appreciate in particular that to argue the rhetorical use of mathematical (or quasi mathematical) statements to support arguments might be regarded as contentious. However I do not see why this is problematical if the usage provides emphasis to an argument that is/or could be both logically and mathematically valid (even if incomplete). I do see it will be regarded as problematic if such usage were used to support unsound arguments. But it should not be as I am concerned primarily with practice. When mathematics is done in practice, it is often wrongly executed. Perhaps the key distinction for rhetorical use of *workplace mathematics* is whether or not the activity forms or is intended to form part of a logical argument (albeit incomplete or wrong).

Do I have any evidence to support these perspectives – the creation of a fragment of a proof/model and the rhetorical usage of mathematical language – as something that Eric could subscribe to as being essential to his construction of narrative? I returned to my observation notes and the evidence gathered as I watched Eric. As Eric read the Bank review and drafted the 'Update on credit position', he built up a narrative with rhetorical force; in particular as he comes across information in the Bank review relating to the Junior Loan notes he adds to his text. He also ordered the text to create coherence. He could have created a reconciliation of the Junior debt outstanding starting with the figure issued to Wallaby as a result of the 2003 reconstruction to expected position at 31st March 2004 if he had wanted to; there is sufficient information in the Bank review to do so. He did not do so. Eric's choice of text and actions provide some evidence that the process is *workplace mathematics* in action. When Eric reviewed the first draft of his text, he looked again at both the Bank review and the first quarter results and he told me that he was "checking the reasonableness of repaying [the] Junior debt." Thus there is evidence in Eric's work practices themselves for the position I am proposing, namely the creation of a fragment of a mathematical model and the rhetorical use of numbers and words that carry connotations of mathematical operations incorporate some *workplace mathematics*.

On the other hand review of the evidence relating to the creation of Paragraph F suggests otherwise, aside from the fact that it contributes more information to the repayment model fragment. Here Eric merely précised the possible future outcome as reported in the Bank review and the inference implied by "this improved performance…." does not follow from his text.

9.2.4 Eric's comments on the trading position – paragraphs D and E.

Exhibit 1b – Paragraphs D and E from exhibit 1

C) ……………

D) Wallaby have set the following goals for their 2004 budget:

- gross margins of £[*sic*] 27% *[improvement implied]* by March '04
- operating costs below £425m by March '04
- positive operating cash flows for 2004 as a whole.(6)

E) They are forecasting sales to increase quarter on quarter, with total sales of £1.6bn in FY2005 (y/e March). (7) Operating cash flows are expected to be positive (pre exceptionals) and overall cash generation is ahead of schedule due to disposals and improved working cap[ital] m[anagemen]t.(8)

Paragraphs D and E are based upon information from the 'Looking forward' section of the Bank review. This comprised 18 lines of closely typed text. Paragraph D slightly shortens the second sentence of the review. While in paragraph E the first part of each sentence is an inference drawn from the text, with the second part being information drawn from the text to justify the inferences partially.

The choice of sentence in paragraph D is interesting. What was Eric's purpose in selecting it? To signal that Wallaby has set goals and (unstated in Eric's text) are implementing plans which will make progress towards these goals. To signal that Wallaby is taking steps to improve its financial position. Also implicitly to signal that gross margins are to be increased and that these and reductions in costs will lead to improvements in profitability and cash flow. All these interpretations are supported by other statements in the 'Looking forward' section. I think Eric intended both Cliff and Tom to use their knowledge and experience to interpret the text in all these ways. The text is effectively a form of shorthand to signify an improvement scenario. Using the text to generate the scenario involves, amongst other things, using accounting models which incorporate *workplace mathematics.* For example one may use the mathematical model of the profit and loss account to reason as follows; if the goals of increasing gross margins (difference between sales revenue and direct cost of sales) and of reducing other costs are achieved both gross and net profits will increase (as sales increasing or static) and operating cash flows will improve, on the assumption that working capital remains the same or is reduced. The information in paragraph E provides such reasoning. Eric's choice of text is excessively parsimonious. Paragraph D is intended to trigger deductions on the part of the reader. Eric may have made similar deductions when composing the text. So was its creation an example of *workplace mathematics*? Maybe. Maybe not. I do not have enough evidence to conclude one way or the other. Eric included the paragraph in his text to indicate that Wallaby had and were implementing a business improvement plan. On the other hand a reader of paragraphs D and E is offered enough information to deduce that Wallaby's profitability and cash flow may improve over the short term; any such reasoning would almost certainly involve the use of the profit and loss model and hence *workplace mathematics*. Thus Eric offers his readers an opportunity to do *workplace mathematics*.

This is to be contrasted with paragraph E. This paragraph provides information which contributes to building the picture of an improvement scenario set up in paragraph D; the paragraph itself is reportage. However in creating the text Eric makes two inferences. First, the statement "They are forecasting sales to increase quarter on quarter, with total sales of £1.6bn in FY2005 (y/e March)" is an inference from the following information in the Bank review:

- "Wallaby ....believe that trading has stabilised;
- sales of £367m in 1Q04[1];
- current market guidance is 2Q sales flat to slightly up;
- for 3Q and 4Q forecast sales both to be between £380m-400m; and
- sales for year ended 31st March 2005 are targeted to be £1.6 billion."

The conclusion "sales to increase quarter on quarter" does not follow clearly from the information in the Bank review; arguably it is not even plausible. It would have been plausible to conclude that sales are forecast to increase in the second half of the year and that sales for the following year are targeted to be £1.6 billion. Eric must have done some *workplace mathematics* but came up with a conclusion which was too optimistic given the information in the Bank review. Second, the statement "Operating cash flows [for 2004/05] are expected to be positive (pre exceptionals)" is an inference from the following information in the Bank review:

- "[1Q] was Wallaby's third quarter of positive operating cash flow before exceptionals;
- they expect to be operating cash flow positive (pre-exceptional).........4th consecutive quarter; and
- in 3Q and 4Q,........continuing to be operating cash flow positive".

The conclusion is a reasonable conclusion, assuming that the forecasts turn out to be true. In making both these inferences Eric used trend analysis where the unit of analysis was quarterly totals. Both of Eric's deductions from the Bank review are examples of *workplace mathematics*. The last clause of paragraph E is reportage only.

In creating this piece of text about Wallaby's operations, Eric has chosen or created text that builds a picture of an improving scenario. In exhibit 1b, I have underlined Eric's words which refer to or carry connotations of mathematical operations; all contribute to a positive picture. Here we have another, not entirely appropriate as explained above, rhetorical use of numbers and words carrying connotations of mathematical operations to create a picture of an improving situation. Eric does not mention in these two paragraphs the negative perspectives which are mentioned in the Bank review; he does, however, refer to them later in his paper – see above.

[1] '1Q04' means the first quarter of the financial year ended 31st March 2004.

9.2.5 Interim conclusion

Superficially the text in the update section of Eric's working paper appears to contain little *workplace mathematics.* That would be the case if he had merely 'cut and pasted' text from the Bank review. Detailed analysis has shown that he has constructed a fragment of a mathematical model and made two inferences of a mathematical nature from information in the text probably using some form of trend analysis. Such work incorporates the use of *workplace mathematics.* He has also created text which readers (particularly those who might use the text to make assessments and judgements) can use to make plausible deductions which would involve the use of *workplace mathematics.* Eric may have done so when he created the text in question but as I have written earlier I am unable to be categorical about this one way or another. But he does more with his text; he uses numbers and words carrying connotations of mathematical operations to warrant his statements and to give his text rhetorical force by creating a picture of an improving scenario. I suggest this is *workplace mathematics* where the activity forms, or is intended to form, part of a logical argument (albeit incomplete or wrong).

9.3 Flawed analysis of half year results

Exhibit 2 – Extract from Eric's working paper

How are the Bank going to get their money back?

The current level of debt is low in comparison..........

[Commentary on Q1 results – similar but slightly less detailed than below.]

Interim results to 30 September 2003 have been subsequently *[the day after the paper was first drafted]* been released. Turnover is down to £756m from £1,019m in H1'02. Operating losses are down at £162m compared to £594m for the same period in the prior year. The improvement is mainly the result of the large exceptional and goodwill charges raised in 2002. The loss before tax also improved to £100m from £914m in H1'02.

T[angible] F[ixed] A[sset]s were £192m at 30 September 2003; cash was £772m; creditors < 1yr were £627m; and creditors > 1yr were £660m. The debt reductions are significant following the restructuring as explained in the prior review on ref:__________.

The cash outflows from operating activities have also improved with operating cash outflows down to £44m from £323m in H1'02. The total cash outflow was £352m mainly as the result of the cash element of Scheme consideration of £340m as described above.

The total level of net funds at 30 September was £99m, due to large cash balances and low debt.

Footnote: I have added the underlinings; the continuous line signifies improvements cited while the dotted lines mark possible problems with the improvement scenario that Eric is attempting to convey.

The above commentary was drafted by Eric in about 15 minutes as he read the published interim results for the six months to 30th September 2003.

The text is a very superficial commentary on the half year results. The commentary is not just a précis of the results but a narrative on the H1 results. The first paragraph sets out the trading position, comparing the results with those of the prior comparable period in the previous financial year. The second paragraph provides information on balances in the September balance sheet which Eric considered to be significant from the credit review viewpoint. The third and fourth paragraphs set out some information relating to the cash flow for the six months and the net funds position at 30th September 2003 respectively.

Composition of this paragraph clearly involves the use of some *workplace mathematics,* mostly through setting up of comparatives of numbers in the accounts for the six months ended 30th September 2002 and 2003. I do not propose to work through the commentary line by line digging out the *workplace mathematics* but to consider whether the text is fit for purpose. The purpose of the commentary is to aid the assessment of how the Bank is going to get its money back – see heading. Is the analysis appropriate? No. It might have been if Wallaby had not been subject to a major reconstruction in the six months under review. The reconstruction resulted, amongst other things, in a write off of debt of the order of £3.5 billion and the creation of junior and senior debt. Also at the time of the write off Wallaby had been left with substantial cash balances to enable it to fund reconstruction and there was an understanding that parts of the business would be sold and the proceeds used to repay some of the debt. During the year ended 31st March 2004 the trading operations, which were to be continued, had contracted and needed to be stabilised. Eric was aware of much of this background – derived from prior knowledge and the reading of the Bank review – but none of this informs his analysis or commentary.

So what happened? Eric followed the practice that he had developed as he performed the loan loss reviews. He assumed that his standard analysis would be fit for purpose; he assumed that it would provide a reader with sufficient appropriate information to assess whether Wallaby was likely to be able to repay its loans (net of the cash collateral provided by it) out of cash generated by its operations and asset realisations. The comparisons provided were neither appropriate nor valid without further explanation because of the radical nature of the 2003 reconstruction. Furthermore Eric coloured his analysis, that is added rhetoric, by painting the scenario as that of improvement. As can be seen from the dotted underlinings in exhibit 2, a picture of improvement is not entirely justified by the facts reported. Eric did not assess the results of this analysis or those of the first quarter.

Eric's process ended up with a paper that was perceived by Tom to be inadequate. During his review of Eric's Wallaby file Tom said to me " It's descriptive. [You] don't get the whole picture" and wrote on his list of review points "Bal Sheet, P&L + cash flow for review please". Tom told me what the team needed to do at the start of his review of some of Eric's files. "The key thing to analyse is where the cash is coming from..........There are four things to look at. First the balance sheet, are there any assets to cover our debt? No. Second the cash flow, is it improving? Are there any improvement plans and has the Bank looked at them? Third, could they sell something – businesses or property? What evidence supports this approach? Fourth, the mug's option. Will anyone take it on? This last option is at the very least questionable."

So why did the analysis go wrong? Again we see Eric using numbers, comparisons and words carrying connotations of mathematical operations rhetorically, and leaving the reader to draw conclusions that require the use of mathematical practices. However the analysis is not fit for purpose. I think there are a number of contributory reasons:

- Both Eric and Cliff started with a preconception that no provision was needed against the Wallaby loans because it had recently been reconstructed. Both told me that Wallaby was not problematic.
- The Wallaby paper was one of the last to be written by Eric. By this stage he had developed a routine (a practice) and that as the loan was perceived to be unproblematic he followed the routine and did not reflect upon his results as he did so. By this time Eric was also "fed up" with the review. (Source: conversation with Eric at end of the audit.)
- I believe that Eric perceived the review to be collaborative, which was in fact the case, and that his job was primarily to write a paper and create a file that enabled the review and secondarily to make a tentative assessment. Tom expected more; he was looking for Eric and Cliff to make firm assessments.

What is the significance of this for the practice of *workplace mathematics?* Eric's performance here raises an important point with respect to *workplace mathematics*. It is a practice and hence involves questions of learning and competent performance and is not just about usage. Eric's work falls apart because he did not understand how to carry out an analytical review of Wallaby's accounts that was fit for purpose. This is not unlike A-level mathematics students who have some understanding of a topic but are unable to do exercises satisfactorily because their understanding is not good enough. To carry out an analytical review of accounts, one has to interpret the results shown in financial accounts to answer questions posed through the use of appropriate comparisons, ratios and other calculations. Investigating the use of *workplace mathematics* inevitably raises questions about competence in performance. In the next chapter, I consider the issues of learning and indirectly competence in some depth and I discuss how Tom enabled Eric to improve his practice.

9.4 Written texts – *workplace mathematics* used in creating written texts

The analysis of an unsatisfactory narrative text has provided a number of insights into the practice of *workplace mathematics*. In this chapter numbers, some mathematical operations and words that carry connotations of mathematical operations play a key role in narrative created in a text. However the nature and extent to which that role involve the practice of *workplace mathematics* is not at all obvious. Detailed analysis shows that some usage is. What is equally interesting is the nature and variety of usages revealed. Classifying some as *workplace mathematics* will, no doubt, be regarded as controversial.

Use of a model fragment and logical inference Eric constructed the narrative by selecting extracts from the Bank review and analysing Wallaby's results. In making his selections, Eric made sense of the texts read and in doing so must, given their nature, have done some *workplace mathematics*. In the course of creating his narrative, Eric constructed the fragment of a proof/model, made inferences using some form of trend analysis and attempted to perform an analytical review of a set of accounts. The construction of a fragment of the proof/model was probably sufficient for the purposes of the loan loss review; Eric certainly considered it to be when he reflected upon his text when he reviewed the first draft of his paper. All these practices involved doing some *workplace mathematics*.

Providing opportunities for others to reason Eric did not complete the analysis of the Wallaby's position in his draft working paper. In certain places he leaves his readers to complete it for themselves if they so wish. So here we have a particularly distinctive usage – the creation of a text to facilitate the reader's reasoning, reasoning that will undoubtedly include the doing of *workplace mathematics.*

Rhetoric and warrant Numbers and words that carry connotations of mathematical operations are often used to gives the text rhetorical force. In the two examples discussed in this chapter, they are often used to aid the presentation of Wallaby's position being that of improvement. The practice is similar to the use of some statistics but is both less precise and less rigorous. Like the popular use of statistics, the opinions expressed are sometimes not justified by the facts/opinions on which they are based. This is a key finding. Numbers and quantified measures are also sometimes used to justify (warrant) statements. I am not suggesting that all rhetorical use of numbers is *workplace mathematics*. I consider that it should only be so regarded where, it is or is intended to be, used to support deductive or inductive reasoning.

Doing *workplace mathematics* unsatisfactorily The unsatisfactory nature of the text reveals another key aspect of doing *workplace mathematics*, not doing it satisfactorily. I am concerned with practice, so this is still *workplace mathematics*. In one case it involved drawing the wrong conclusion. In another it involves making an imperfect selection of material to include in the text. In the case of the analytical review of the accounts, it included not knowing how to carry out the review effectively and not reflecting on the results revealed.

Detailed analysis of a piece of text using the text, my observation notes and the underlying texts has considerably enriched the meaning of what it is to do *workplace mathematics.* It highlights the importance of a focus on the process of what is done and, through unsatisfactory performance, directed attention to the importance of learning and reflective practice as one works. I now turn to consider these issues in depth.

10 The role of *workplace mathematics* in on the job learning: understanding situations and learning to do

10.1 Introduction

My participants often had considerable freedom as to how they carried out tasks in hand. The task might be set: to review a set of accounts (Holly); to construct a more detailed goodwill model (Joan); or to do an agreed list of loan loss reviews (Eric). However most of the tasks observed required much more than the repetition of well practised routines applied to new information gathered in an appropriate form. They required the participants to understand the facts and circumstances relating to particular events/situations and to assess the reasonableness of the company's treatment of them from an accounting perspective. The requisite learning involved covered a wide range of activities ranging from deriving meanings from tables showing financial information to learning how to carry out an analytical review of a company's accounts more effectively.

Learning, itself, in the workplace is often not an explicit object; it enables one to do one's job. Explicit work related learning often involves studying to obtain necessary or desirable qualifications, or is on the job or near the job training, which enables individual or team performance or provides knowledge and understanding of a firm's systems. (Source: discussion with Beatrice, one of the managers on the Bank audit.) In this chapter, I analyse some of the participants' work practices to illustrate the role *workplace mathematics* played as they learnt. I set out to find out if and how my participants learnt *workplace mathematics* on the job and in the process discovered that *workplace mathematics* itself was a key technique which was harnessed to facilitate learning. In the next chapter, I consider reflective practice and innovation.

In this chapter I use some of my data to determine the role of *workplace mathematics* in the learning observed. The learning was all determined by the need to carry out a particular task, the particular incidents being situated in particular facts and circumstances. The techniques used were less local. It is the techniques used rather than the specifics of a particular case that are of interest here. The techniques observed are discussed under the following headings:

- reading to extract meaning;
- using exploratory calculations;
- asking questions;
- using partially built models;
- learning facilitated by the organisation of the auditing processes; and
- teaching and learning.

Some of the episodes have already been described in detail or form part of the background to the practices discussed and therefore are not described in much detail here.

10.2 Reading to extract meaning from tables and schedules produced by others; learning to enable a task

10.2.1 Introduction

In this section we encounter Gary and Ramesh, two student accountants, auditing the calculation of specific figures in the draft year end accounts. We see them reading tables and spreadsheets: firstly to gain understanding, i.e., to learn about a situation in order to carry out a designated task; and secondly to carry out parts of that designated task. The composition of the numbers and the related mathematical structures are at the centre of their learning. We encounter them struggling to understand numbers and their intended meanings and using *workplace mathematics* to aid their learning. The examples were chosen as it was reasonably clear from the evidence collected that the focus had to some extent be upon the numbers and mathematical structures.

10.2.2 Gary understands the schedule of directors' remuneration

Gary, a second year accounting student, was asked by his in charge, Clare, to check the veracity of the information relating to directors' remuneration in the notes to a set of accounts. This was a task he had never carried out before. Clare gave Gary the client's schedule showing the composition of the directors' remuneration and left him to get on with the task. For more detail see subsection 5.2.3. The first thing that Gary did was to look at/read the table. He looked at it intently and continued to do so as he checked the totals of all the columns and rows with a calculator. He then checked the main grand total and the details of the managing director's remuneration to the figures in the draft accounts, and started checking details in the summary schedule against other information/records demonstrating that he had understood the schedule.

Given the intensity with which Gary looked at the table as he meticulously checked all the calculations in the table, I believe he was buying time as he absorbed the contents of the table so he could work out what to do. His manner was normally more relaxed when he used his calculator despite the fact that most of the calculations performed were more onerous. He was learning as he focused on the mathematical structure of the table and of its contents; there was not anything else to look at. I believe he may have being doing several things all at once, all of which to a greater or lesser extent were likely to involve mathematical activity, i.e., *workplace mathematics*:

- seeking to understand the row/column structure of the table including both its commercial meanings and mathematical structure;
- seeking to understand the information in a typical row;
- deciding which totals to check over to the accounts;
- thinking about how he might audit some of the figures in the table; and
- checking the arithmetical accuracy of the table.

Gary could not have taken his designated task forward without understanding the mathematical structure of the table and its contents. I believe his focus was in making sense of the information in the table, the structure of the calculations and all the numbers, and not the calculations *per se*. *Workplace mathematics*, thus, played a key role as he learnt about the directors' remuneration.

Gary may also have assessed the reasonableness of the information in the table. As there was nothing untoward, I have no evidence to comment on whether or not this was the case.

Reading and understanding financial information, tables and the detailed results from models was a prerequisite of many of the auditing tasks carried out by my participants. For example, Cliff learnt in a similar manner when he reviewed the Kookaburra debt valuation. Considerable learning occurred when participants read documents carefully for the first time, particularly where the reader was relatively inexperienced. The learning was directed towards making sense of particular situations/events. As much of the information discussed/represented in the texts used was in financial form, *workplace mathematics*, as demonstrated above, was used. However, the role of *workplace mathematics* is more; it enables learning itself, as is demonstrated by the next example.

10.2.3 Ramesh assesses a vacant space provision

a) Background

Ramesh, a final year student accountant, worked for four weeks on the Bank audit. One of his tasks was to audit the Bank's provisions for future costs relating to rented offices which are no longer used in the banking business, i.e., the vacant space provisions. Part of the task involved assessing the reasonableness or otherwise of the calculations of large provisions relating to specific properties. By the time I came to observe Ramesh, he had already had a meeting with the client to discuss the provisions. I now consider Ramesh's learning as he conducted his initial review of one of the large provisions prepared by the Bank's property advisers. A subsequent review of another similar provision followed a similar course but took less time reflecting Ramesh's learning. The dilemmas raised in the first part of the review, which are discussed here, were resolved in the second part of the review, which is discussed in section 10.3.3 below as it illustrates another learning device.

A vacant space provision is calculated by estimating the net present value (NPV) of the quarterly cash outflows likely to occur during the remaining term of the Bank's ownership of the lease. The cash flows for the leases under discussion were all discounted back to 31st December 2003. The calculations and valuations were carried out on an Excel spreadsheet. For each quarter of the unexpired term of the leases, the quarterly losses were calculated. Each loss comprised the rent payable to the landlord under the headlease less the estimated income receivable from subletting plus the estimated expenses payable. The expenses were analysed across a number of headings. Where the income and costs were not certain, the Bank together with their property advisers made assumptions to enable income and expenditure to be calculated, e.g., assumptions were made about refurbishment costs, subletting periods, rentals and costs relating to void periods. Beneath the total for the loss for each December quarter, the NPV of the annual loss is shown. The figures, including the NPVs, were totalled and set out in the second column of the table, while the details of the undiscounted quarterly cash flows were in the next 40 columns, as the longest lease had a 10 year term remaining.

b) The work done by Ramesh

Ramesh started the substantive work on the provision by spending time trying to make sense of the valuations produced by the Bank's property advisers. The valuation comprised the two page forty two column Excel spreadsheet described above and two tables specifying the assumptions on which the valuation was based. Ramesh spent 12 minutes reading and checking the assumptions schedule, only 2-3 minutes looking at the NPV calculation on the Excel spreadsheet and 10 minutes trying to work how the estimate of the income from subletting the ground and first floors of the property was calculated.

Table 1 – Property data – extract from the schedule of assumptions

Building	Total area	Effect ive area	Passing Rent*	Effective Rent discount	Current Office ERV	Headline ERV**	Effective ERV payable	Rever-sion
	(sq. ft.)	sq. ft.	(£pa)	(i)	(£psf)	(£pa)	(£pa)	
Floors 3-4	16,708	16,708	283,034					-!00%
FloorsG-1	24,178	24,179	478,240	27%	£12.50	302,225	220, 624	-54%
	(1)	(2)	(3)	(6)	(13)	(14)	(15)	(16)

Footnotes:

Headlease for floors 3-4 terminated on 30/6/04 and the remaining term of headlease for floors G-1 was 10 years.

'ERV' means estimated rental value or equivalent.

Numbers in red indicate location of column in the original table.

*annual rent payable by Bank

**estimated rent receivable by Bank from subletting

Ramesh's learning process was both messy and time consuming, possibly because he had a lot to learn. As a consequence so is this description of his review of the assumptions schedules. Ramesh's approach to understanding the valuation did not involve briefly reading through all the papers but started with him homing in on the information in the assumptions schedule that he thought would be key, leading him initially to overlook the small table which set rental growth assumptions for the subletting of the property.

Ramesh began by comparing the assumptions in the year end schedules to those in the half year schedules "to see if [there are] any major differences". As he read he put his fingers under the numbers in the above Property data table and moved them along as he looked to and fro between the two tables of assumptions. Ramesh's pursuit of meaning centred around the numbers – the lettable area (2), the rent payable to the landlord (3), dates, the frequency of rent reviews etc. In this process he used *workplace mathematics* (in italics) to facilitate his understanding. The numbers and their meanings were put at the centre of Ramesh's learning. I consider that Ramesh drew meaning from the half year schedule beyond *his stated purpose of looking for major differences.* As he read along the numbers in table 1, he checked (audited) them against the half year schedule, resulting in the checking of numbers being intertwined with the process of understanding. *He also made mathematical connections between the assumptions*; *he calculated the rent payable per square foot to the landlord* (e.g., £478,240 **(from 3)**/24,179 **(from 2)** = £20 per sq. ft.), *he checked the calculation of the rental income from subletting (14) from other assumptions* (rent per square foot (13) * lettable area (2) = £12.5*24,178 = £302,225 (14)) and he almost certainly made other connections but these were not made explicit to the observer. Ramesh explored the mathematical relationships between numbers both to gain and/or check his understanding of their meanings and to create new meanings.

Ramesh then started to synthesize the information in the table to try to construct relevant meanings as is revealed through his actions and his comments to me; these comments have a mathematical comparison (*workplace mathematics*) at their core. When Ramesh finished reading the assumptions schedule, he made some notes at the foot of the schedule, and commented on some of the connections he had made. He told me that "It looks as if the rental values have fallen from £20.00 per square foot to £12.50 per square foot. This is bigger than expected. 10% was predicted last year." He was not wholly satisfied by his explanation; he compared rents for the Lyrebird property with those for another property and thought about the comparative falls and said in a hesitant tone "OK OK makes sense" but clearly was not happy that it did. In fact Ramesh had both used the wrong percentage (he takes percentage from the half year rather than the year end schedule) and misinterpreted the meaning of rent growth percentage (it referred only to income from subletting and not to the relationship between the rent payable to the landlord and income receivable from subletting). He subsequently cleared up these misunderstandings when he reconciled the rent shown as receivable from subletting in the NPV calculation to the information in the Property data table – see subsection 10.3.3 below.

Ramesh moved onto the NPV calculation. He did not spend much time understanding it. He was familiar and comfortable with the principles of discounting to obtain NPVs and spreadsheet calculations of specific examples.

Ramesh like Gary read the papers in order to understand, i.e., to learn about, the NPV calculations. He also checked some of the information in the table. In doing both he used *workplace mathematics* as much of the information in the papers was mathematical in nature. But that is not all. Ramesh concentrated on that with which he was not familiar. His focus was on the assumptions that he assumed to be key and their related mathematical structures and meanings. And as can be seen from the above *workplace mathematics* was pivotal to Ramesh's sense making. *Workplace mathematics* facilitated his learning.

10.3 Using exploratory calculations to aid sense making

10.3.1 Introduction

Ramesh used many exploratory calculations as he worked. An exploratory calculation is a calculation made by a participant to aid understanding of a text or a situation. It is an activity which takes place as a participant works to understand information produced by others, including computer printouts. The reason for carrying out the calculation varies: it may be used to structure thinking as a participant struggles to make sense of a situation; it may be made to determine or to confirm the mathematical structure of a figure; it may be made to determine how a figure is made up or how it reconciles to another; or it may be used to generate new results that provide further insights into the meaning of particular situations. Exploratory calculations, which by their nature are *workplace mathematics,* are made to aid learning and sense making.

10.3.2 Exploratory calculations; the practice described

Throughout my fieldwork I observed participants using this type of calculation as they strove to understand particular facts and circumstances. Certain distinctive characteristics seem to attach to them. Most were performed on a calculator. As a participant tries to understand how a number is composed, s/he reaches for a calculator, which has often been deliberately laid on her/his desk so it is readily available, and rapidly punches a calculation into the calculator and looks at the answer. If the answer is satisfactory, s/he may move on, write an explanatory note in the working papers or indicate that is it is so by saying something like "We're there". If the answer is not satisfactory, the participant may carry out further quick calculations, which tend to follow on rapidly one after the other – this may or may not produce a satisfactory answer. Alternatively

- s/he may reflect and decide what to do next;
- s/he may continue reading and seek to resolve the issue as s/he gains more knowledge;
- s/he may ask questions of the client or other team members; or
- s/he may spend time trying to resolve the issue satisfactorily.

Sometimes a line of thought is abandoned. Other distinctive features are: the calculations are performed more rapidly than those made to check the accuracy of workings; initially they tend to be done for oneself and only shared if successful (this had the consequence that often I was unable to record the actual calculation made – I can sometimes reconstruct the calculation from other evidence); and they tend to be done when a participant is working alone. Exploratory calculations are powerful flexible tools that aid learning and sense making. Ramesh's practice as he worked on the Lyrebird provision illustrates this.

10.3.3 Ramesh uses exploratory calculations

The exhibit and table need to be read in conjunction with the text that follows.

Exhibit 1 – Extract from my observation notes of Ramesh

	Checks e-mail "We are going to be working on City provisions" Shows me his working paper 'Vacant Space Provision (City)' ……….
9.25am	…….. Turns to … the KKL *(property advisers)* papers for the Lyrebird property *(These comprise an Excel worksheet with the cash flow and the results of discounting calculations and schedules setting out the assumptions on which the calculations are based.)* *Ramesh checks the assumptions schedule. In the process he carries out **four** exploratory calculations.* ………..
9.37am	"OK OK makes sense" ………… He then turns to the cash flow spreadsheets Checks void periods properly reflected in spreadsheet Checks rent free periods properly reflected in spreadsheet Starts checking/understanding other numbers in cash flow, paying particular attention to the rents from subletting** Thinks about the £70k quarterly rents receivable *(income from subletting)* on page 2 of cash flow

Does exploratory calculation (5)

Reads note at the bottom of the first box on PV842/3** *(main assumptions schedule)*

Reads page 2 of PV842/3

Then reads page 2 of cash flow and says to himself

"OK"

Looks at cash flow

"Right cash flow looks appropriate looks reasonable

"70 is approximately 75 (6) and 302/4 (7) = 75

"70 is reasonable having regard to views expressed about future rents"

9.40am "Got to show it is reasonable"

Uses calculator lid as a ruler

Makes note on lead schedule re void period. And then references it.

Flicks through papers**

Looks quite hard at cash flow

"Fine"

Gets calculator and does exploratory calculation (8)

302*0.85 =256/4 = 64

"Question is how do we show"

Highlights numbers on front sheet in green.

Notes what green means saying as he writes

"We have 302k, headline rent receivable, less 15% per assumptions = 256k. Therefore £64,223 per quarter."

"It does seem reasonable"

"Now we need to calculate NPV"

9.50pm "Right"

Note: Between 9.25 am and, say, 9.45 am, Ramesh reads that rental growth for 2004 is assumed to be minus 15%. This information is in the table at the top left hand corner of the assumptions schedule. He did not pick this up when he first read the schedule and I speculate that he did so at one of the three points marked **. This is a clear illustration of the iterative nature of learning.

Table 1 – Property data – extract from a schedule of assumptions

Building	**Total area**	**Effective area**	**Passing Rent**	**Effective Rent discount**	**Current Office ERV**	**Headline ERV**	**Effective ERV payable**	**Rever-sion**
	(sq. ft.)	sq. ft.	(£pa)	(i)	(£psf)	(£pa)	(£pa)	
Floors 3-4	16,708	16,708	283,034					-!00%
FloorsG-1	24,178	24,179	478,240	27%	£12.50	302,225	220, 624	-54%
	(1)	(2)	(3)	(6)	(13)	(14)	(15)	(16)

Footnote:
Headlease for floors 3-4 terminated on 30/6/04 and the remaining term of headlease for floors G-1 was 10 years.

Ramesh performed four exploratory calculations during his initial review of the assumptions; the first two as he read columns (1) to (3) and the second two while he was concentrating on columns (13) to (15). All were performed on his calculator.

The first two involved calculating the rent per square foot for the passing rents (i.e., the rent payable by the Bank); they were £17 and £20 per square foot. What did Ramesh gain from the exercise? Generally commercial rents are quoted as being so much per square foot. Thus Ramesh was able to compare the rents payable in respect of Floors 3-4 and 1-G with each other and with those for another property. The calculations deepened his understanding and provided him with new information.

As a result of the third and fourth calculations, which were performed rapidly in quick succession, Ramesh found out that rent per square foot (13)*effective area (2) = estimated rent receivable from subletting (14), i.e., £12.5*24,179 = £302,225. These calculations were confirmatory; they added no new information. However Ramesh considered the result to be sufficiently important to note it in detail at the bottom of the main assumptions schedule.

During the course of looking at the cash flow, Ramesh performed four more exploratory calculations, two mentally and two on his calculator – see sections highlighted in exhibit 1 in bold type. They all relate to his attempts to understand how the estimated rent receivable from the subletting of floors G-1 was generated for the cash flow. He had already tried to do this as he read the assumptions schedule – see subsection 10.2.3. In the cash flow it is assumed that the floors would be subject two subleases that run consecutively and that the quarterly rents receivable would be £64k and £70k respectively. Ramesh considered the calculation of the rents when he was looking at page 2 of the cash flow. He compared the quarterly rent payable (£70,000) with the annual rental figure (£302,225) in the assumptions – exploratory calculation 5 was performed during this process (I have no record of that calculation); it seems Ramesh was not sure how they related to each other. He then re-read parts of the assumptions schedule, looked again at the cash flow, and mused aloud to me making two further exploratory calculations: "Right. Cash flow looks appropriate. Looks reasonable. 70 is approximately 75 and 302/4 is approximately equal to 75. [Further] 70 is reasonable having regard to views expressed about [the decline expected in] future rents". Although Ramesh commented that the figure looked reasonable, he was clearly not satisfied as I believe he thought the cash flow numbers should reconcile precisely with the assumptions. In fact he was not even looking at like for like – the £70,000 rental figure related to the second sublease and the £302,225 was the 2003 estimate. He read the papers yet again, looking at cash flow carefully. During this process he noticed that the assumptions predict that there will be a 15% decline in rental values in 2004. He used this information to perform two exploratory calculations as follows: 302*0.85 = 256 and 256/4 = 64 (workings in thousands) being the quarterly rent expected under the first sublease. This agreed to the figure in the cash flow. Ramesh after considerable work understood the position and wrote up an explanation on his working paper, saying, "It does seem reasonable".

Ramesh used explanatory calculations to aid his sense making in a number of different ways:

- they generated new information, the rent payable per square foot, which in turn enabled the making of comparisons;
- they confirmed the validity of the calculation of rent from subletting in the assumptions schedule; and
- they played a critical role in resolving his problems in understanding how the rents from subletting were calculated in the cash flow.

My other participants used calculations in a similar way. For example, Gary made extensive use of exploratory calculations to structure his inquiries into the nature of the managing director's annual pay and to determine its composition. Eric spent 2 hours creating the table to summarize the Bank's loans to Koala; there were two differences (and substantial differences in presentation) between the information on the CLM and one of two facilities summary; there were 31 balances listed in the CLM. Eric carried out five exploratory calculations as he tried to reconcile the numbers. He also made 7 amendments to sums inserted in formula cells, 4 being made during the exploratory period and he found and corrected two errors, one being of input and the other using the wrong foreign exchange rate. I also observed him use them from time to time as he worked on the other loan loss provisions. Tom used three to test his expectation that Kookaburra would have to write off about 50% of its debt. Exploratory calculations, which by their very nature are *workplace mathematics,* are powerful aids to learning and sense making, particularly where mathematical meaning or structure is not clear to a reader.

10.4 Asking questions

My participants tended to ask questions to clarify situations when they did not understand something. Both Gary and Ramesh asked other team members. Cliff and Tom learnt in review meetings through questioning. All asked the client. Once Eric completed the first draft of a loan loss review working paper, he usually had a list of queries for the Bank's loan relationship manager. The whole purpose of Sasha's meeting with Sonjoy, the client, was to go through a list of outstanding items so she could progress her work. Given the nature of the information subject to audit, questioning inevitably involved some *workplace mathematics.* Although there was no discussion of mathematical modelling techniques in the limited discussions observed between participants and clients, it is clear from other evidence that clients do explain how mathematical models work where they are used to generate or justify provisions in the accounts, e.g., Ramesh's pre-observation discussions with the client and Joan's meeting with a client following her first revision of the goodwill model.

10.5 Using the results from partially built models

In chapter 8, I have dealt extensively with models and *workplace mathematics.* Here I make one point with respect to learning. For my participants extracting meaning from papers describing or recording events was an iterative process. That process often carried over into the building of mathematical/financial models.

Models when they were in the process of construction were used to identify where information available was insufficient or a version of the model was not yet fit for purpose. Sasha told that me that she constructed a spreadsheet for each item in the stock/programme accruals tests at a relatively early stage of her information gathering in order to see what she had got and by implication what she still needed. When Joan's revised version of the goodwill model showed large losses, she in conjunction with her manager decided that she needed to talk to the client to understand more about the facts and circumstances and the assumptions that underlay the purchase. After Eric build the first phase of the cash flow model for Possum, he decided that its results did not adequately explain how the cash flows generated were used so he extended the model further. Thus participants learnt by using models as they were in the process of construction. All the activities described involved some use of *workplace mathematics.*

Perhaps one of the reasons why participants modelled particular facts and circumstances was to aid the development of their understanding of the particular situation under consideration.

10.6 The institutional organisation of an audit – an environment that facilitates learning

The organisation of audit work facilitates learning. In this section I show through examples how *workplace mathematics* is inevitably part of that learning; all the examples quoted apart from that relating to Ramesh are discussed elsewhere and all involved using, and sometimes developing the skill of using, *workplace mathematics.* Inbuilt to audit work are a number of theories of learning: learning is achieved through doing; practice makes competent and in due course expert; and teamwork together with an open reviewing system creates an active learning environment. The learning includes learning about particular facts and circumstances and learning about the techniques used in work done.

Gary was left to get on with the review of the directors' remuneration note, even though he had never done this before. Eric created the Possum cash flow from information in the company's accounts; it was clear from his hesitant but competent performance that he had not (or had not often) done this before within the workplace. Eric showed assurance and competence in creating the loan summary tables; competence came from practice.

The audit review process is a two way interactive process. Cliff and Tom, as reviewers, learnt about situations and issues by reading and critically reviewing work done. Their understanding was speeded up by up the existence of the draft audit papers, requests to be talked through the issues and the questioning of the in charge, in this case Eric. Discussion in a review meeting, particularly if it is open, allows new perspectives and issues to emerge – as happened in the Kookaburra loan loss review. Managers/in charges/audit assistants also learn from observing the performance of others. Eric and possibly Cliff would have learnt from Tom as he, through example, showed how to reason about the likelihood of recovery of the Bank's loans to Possum using the cash flow constructed by Eric.

Ramesh was not sure how to make an independent assessment of one of the vacant space provisions, so he put it to one side on the first morning I observed him. When Jerome, his manager, came to this provision during an interim review meeting, Ramesh asked Jerome for advice about how to audit the provision. As Jerome and Ramesh looked at the detailed calculations of the provision, Jerome, with assistance from Ramesh, built up his understanding of the subleases of the different floors and the relative size of the parts of the provision and then he advised Ramesh how to proceed calling upon knowledge of where to obtain third party evidence that could be used to support the client's calculations. Here Jerome and Ramesh learnt together as they sought to extract meaning from the provision schedule and Jerome taught Ramesh about how he should approach auditing the provision, but only after he had made partial sense of the make-up of the provision. *Workplace mathematics* was used in the sense making process but not so much in the subsequent teaching.

The Review process often leads the way to better practice and sometimes involves coaching and teaching. After his review of the Possum file, Cliff suggested that Eric should express the relationship between the debt and cash flow as a ratio rather than as years to repayment as it was not realistic to assume that the cash flows would be used solely to repay debt. Tom's review of the draft working paper on the Wallaby loans led to a major episode of teaching and learning, Eric's reanalysis of the Wallaby half year results. This is discussed below.

10.7 Teaching and learning; how to review accounts analytically

In chapter 9, I have written about my dissatisfaction with parts of Eric's working paper on Wallaby. Tom also found it unsatisfactory; as he reviewed it he commented "Fantastically irritating - lengthy descriptions. Why don't [they] put the financials *(meaning the published results)* in the file and cross refer, so you don't have to write on it. Descriptive [but] you don't get whole of picture." But despite this, Tom added "I am sure [that the] conclusion is right."

Tom then had a private informal discussion with Eric, in which he showed Eric how to review a company's accounts analytically in order to assess whether or not the company was likely to be able to repay its loans. As a result of this Eric sat down and typed up a set of notes on how to approach credit reviews (appendix 4) and then reanalysed Wallaby's half year results (appendix 5). I observed none of this.
Tom's intervention enabled Eric to transform his work. Eric learnt by listening to Tom's teaching, writing up his understanding and then using those notes to support his reanalysis of the Wallaby half year results. The extent of this transformation is best illustrated by comparing parts of Eric's texts pre and post teaching.

The exhibits need to be read in conjunction with the explanatory text which follows.

Exhibit 2 – Extract from Eric's working paper on Wallaby – pre Tom's review

> The cash outflows from operating activities have <u>also improved</u> with operating cash outflows <u>down to</u> £44m from £323m in H1'02. The total cash outflow was £352m mainly as the result of the cash element of Scheme consideration of £340m as described above.

The information is extracted directly from the published results. It is very limited and there is no analysis. Eric uses the reported facts as part of his narrative of improvement – see my underlinings. However he does not address the key issue, namely how Wallaby's cash flow might impact on the recoverability of the Bank's loans. This is in marked contrast to Eric's revised paper on the Wallaby half year results – see excerpts on cash flows in exhibits 3 and 4.

Exhibit 3 – Extract from paper on Wallaby's half year results – post Tom's review

> <u>Cash Flow</u>
>
> Net cash inflows from operating activities were £65m, before exceptionals. Before working capital movements the total cash outflow was £21m. The net movement in working capital was £86m, the result of large decreases in stock and debtors. Stock decreased by 14% and debtors by 20%. Meanwhile the level of creditors increased *(should read 'decreased')*. These working capital movements are not sustainable and therefore in future period we would expect the level of operating cash inflows to decrease unless there is an improvement in the level of cash generated profits.
>
> [Paragraph 2 below]

.
Eric describes and analyses the composition of the cash flow in some depth. He generates new facts using calculations and then analyses his findings. The figure for the cash flow before working capital movements, £21m, is calculated as follows: net cash inflow - decrease in net working capital = £65m - £86m = outflow of £21m. The movement in working capital derived from information in note 16 of the results: cash inflow = decrease in stock over six months + decrease in debtors - decrease in creditors = £33m + £123m - £70m = £86m.The decreases in stock and debtors are then quantified as percentages. Creditors decreased, not increased as Eric writes. Eric's mistake is probably inadvertent as the movement in creditors is treated correctly in the calculation of the net working capital movements. The comment that the company is unlikely to be able to continue to generate future working capital savings is a sensible commercial comment and gives rise to the deduction that cash flows from operations are only likely to improve *(and also to be maintained)* if profits increase.

Exhibit 4 – Extract from paper on Wallaby's half year results – post Tom's review

Cash Flow

[Paragraph 1 above]

Post exceptionals the operating cash outflow was £44m *(£65m - £109m = - £44m)*. The £109m of exceptionals can be broken down as follows: ESOP Settlement £35m; Restructuring costs £55m; and "Other" £19m. Net interest payments were £19m. Therefore net operating cash flows post interest but before exceptionals was [*sic*] £46m *(£65m - £19m = £46m)*. This would result in the debt being repaid in 11 years (based on grossed up figure of £92m *(£46m*2 = £92m)* cash inflow), this assumes that the improvement in cash profits mitigates the current advantageous movements in working capital.

The figures in this paragraph are derived mostly from the notes to the half year accounts. Most of the re-constructed calculations performed are annotated on the exhibit. I am unable to work out how Eric got to an 11 year payback period with any degree of certainty. The remarks regarding cash profits and movements in working capital follow from the comments in paragraph 1. Eric's narrative in this text comprises noting the impact of the reconstruction costs *(which are likely to be one-off costs)* and a 'what if' mathematical model. The model is not a model of possible future cash flows of the group; it is a model which calculates a hypothetical repayment period using two assumptions namely that future cash flows are equal to current cash flows and that operating cash flows net of interest payments *(possibly post tax)* will be used to repay loans. In making his assessment, Eric assumes operating cash flows will be maintained and ignores the fact that Wallaby is a going concern and that it is likely to invest part of its cash flow in its business

Tom's teaching transformed Eric's practice. It led Eric to answer directly the key question namely, 'How is the Bank going to get their money back?' As can be seen from the analysis of the comments on Wallaby's cash flow, there is a major qualitative difference in the texts produced pre and post coaching. The review of the H1 results in the working paper is a brief commentary on the results and no more, while the re-analysis evaluates Wallaby's ability to repay its debt. The first text provides a superficial narrative on the H1 results. It reports on the current period results and key figures, and compares the current period results to those in H1'02. The paper on Wallaby's half year results contains a number of quantitative measures highlighting the level of Wallaby's indebtedness, its cash position, and the nature of the cash flows thrown off in the 6 months to 30th September. The measures were derived directly from the accounts or through calculation. Most of the quantitative measures are used to make interpretations about Wallaby's financial situation and its ability to service its debt obligations. Eric used relatively little *workplace mathematics* when he compiled the first text: he extracted numbers from the accounts, including comparatives, and explained how significant numbers contributed to certain totals. On the other hand *workplace mathematics* is central to the second text. For example in the comments on the cash flows, Eric used numbers from the accounts;

- to provide four different measures of cash flow for the six months;
- to explain how those measures were derived;
- to comment on the sustainability of the cash flow from operations; and
- in a mathematical model to make hypothetical calculations; resulting in
- an assessment of Wallaby's ability to repay its debts and hence to repay the Bank.

Eric's response to Tom's intervention was transformational. He took time to write a note setting down his understanding of what Tom had said (appendix 4). He then put the theory into practice, the Wallaby paper being the result (appendix 5). Listing the techniques used by Eric does not do justice to the nature of the transformation. For example as Eric constructed the paragraphs on the cash flow position, we can postulate that he formulated and answered a series of questions, namely what was the operational cash flow, would it sustainable in the future, and if current cash flows were replicated in the future, would the cash flows generated suggest that Wallaby might be able to repay its long term debt within a reasonable period of time. To answer these questions or similar questions Eric:

- extracted information from the published results, this involved some minor calculation;
- set up some other mathematical relationships between the numbers, the purpose being to bring critical information to the foreground; and then
- in each paragraph extrapolates the results putting then into an appropriate commercial context.

As a result Eric produced positive evidence that contributed to the evaluation in early 2004 that Wallaby was likely to repay the Bank loans. His first commentary produced none; if anything it provides negative evidence despite the fact that it has been constructed as an improvement narrative.

A question worth asking is why was this teaching so effective, particularly as I believe that Tom had previously discussed with Eric on more than one occasion how to do a loan loss review. I believe Eric had insufficient practical experience to do the job without help. (Evidence: Cliff's guidance on the use of ratios, relative inexperience shown when constructing the Possum cash flow, and only articulating concerns about Kookaburra when encouraged to do so.) Furthermore I believe Eric did not wholly absorb what was required when he was briefed initially and when Tom provided informal coaching, partly because there was a huge amount of information to absorb and partly because he needed some practice at creating the first drafts of the loan loss reviews to get a feel of how to extract pertinent information from the large volume of papers provided. So why was Tom's teaching session so effective?

- Tom had identified that Eric did not have an adequate theoretical framework to carry out analytical reviews of accounts for loan loss reviews.
- Tom decided to take Eric aside and spent time teaching him.
- Tom, as is demonstrated by other evidence in this thesis, had a well developed theoretical framework and was a good teacher.
- I believe that Tom in his teaching would have both explained the purpose of the reviews and posed some questions that might need to be answered, and was able to describe clearly the mathematical relationships, the models and the commercial explanations that could be used to answer those questions.
- At the time Tom took Eric aside, Eric had more than sufficient practical experience of the reviews such that he was able to absorb Tom's teaching and put into the required context.
- After the session, Eric wrote the paper to capture his understanding of what Tom had talked about – in the process he would have deepened his own understanding. And finally
- Eric put Tom's teaching into practice almost immediately, the practice being work rather than an academic exercise.

10.8 The techniques of learning observed summarized

The tasks undertaken by my participants often required intensive learning in order to get the job done. When I reviewed my data to determine how *workplace mathematics* was learnt, I found several episodes. I also discovered that *workplace mathematics* was used frequently as an aid to learning. *Workplace mathematics* was implicated in workplace learning in three ways.

1) Used as making sense of information Given the nature of the information used and the nature of auditing for financial reporting, my participants used *workplace mathematics* extensively as they read and so learnt about the facts and circumstances of events and situations and audited them. For example in this chapter we see Gary and Ramesh using *workplace mathematics* to make sense of a table and a NPV calculation prepared by clients (subsections 10.2.2 and 10.2.3). Cliff used *workplace mathematics* as he sought to understand the debt valuation model (subsection 8.2.5 b)).

2) Learnt for itself Both the learning of, and the teaching/coaching and learning of, *workplace mathematics* were observed. For example:

- As Eric constructed the Possum cash flow he learnt through doing; he learnt how to construct (i.e., perform) a cash flow calculation, as well as tentatively interpreting it (section 8.4).
- Cliff explained to Eric why he should use debt/cash ratios rather than payback periods in certain circumstances (subsection 8.4.4).
- A major incident of teaching and learning occurred when Tom explained to Eric how to conduct analytical reviews of accounts to assess a group's ability to payback its loans. Eric reinforced his learning by writing up the notes and putting them into practice almost immediately (section 10.6).

3) As a scaffold to aid sense making But perhaps more significant as less obvious was the role that *workplace mathematics* played as an aid to learning. In this chapter, I have highlighted a number of cases where the participants used *workplace mathematics* to improve their understanding of situations.

- Gary and Ramesh focused on financial numbers and their relationships as they seek to make sense of texts as they read.
- They also used exploratory calculations as a scaffold as they struggle to understand the make up of figures that they perceive to be problematic (section 5.2 and subsection 10.3.3). Exploratory calculations are a distinctive and important *workplace mathematics* technique which was commonly used by my participants. Distinctive features of the practice are described in subsection 10.3.2 while its usage by Ramesh is described and analysed extensively in the following subsection. The name, exploratory calculation, derives from the description that I assigned to the practice as I wrote while observing my participants.
- Given the nature of the information that my participants dealt with, *workplace mathematics* was inevitably incorporated in other learning techniques:
    - questioning;
    - evaluating and responding to results of models in the process of building;
    - through carrying out designated tasks; and
    - throughout the review process.

In earlier chapters, I have discussed extensively how participants used *workplace mathematics* to assist in the making of judgements. Thus a key finding of this study is that *workplace mathematics* is a powerful flexible tool which assists learning where *workplace mathematics* is embedded in the information, artefacts and practices with which participants' work.

I now turn to consider how new ideas/practices were introduced as a task in hand was executed.

11 Reflective practice: importation of new ideas/techniques to the practice of work

11.1 Is transfer an appropriate concept?

My participants to a great extent performed and completed the tasks in hand competently or with a degree of competence. Most thinking as demonstrated through observed action (or report) was a direct response to the task in hand and the available resources. The evidence and discussions in this thesis show that thinking and action were mostly specific to the task in hand. The resources upon which the participants called – texts/papers, technology, practice and procedure, routine and routines, prior education and training and on the job coaching – all played a role. Active collaboration within the team was also important. However I consider that thinking and action driven by the specific complex local contexts in which the participants worked was not of itself sufficient to account for competent performance. My participants introduced new techniques and ideas and modified practices to facilitate the execution of tasks. Whether or not this constituted transfer of learning is a moot point. Transfer, namely cross-task generalisation between a problem and its isomorph or its near isomorph, is problematic. A general finding in cognitive psychology literature is that transfer is unusually low; knowing how well participants do in one problem is of little help in predicting how well they will do in an isomorphic problem in another context (Ceci, Rosenblum and DeBruyn, 1999).

Within a situated cognition paradigm, a more appropriate way of looking at innovation/reflective practice is to find out what participants did, i.e., how they innovated, and to use this evidence to consider how it arises. In this chapter, I list a number of incidents where participants introduced something new into the immediate work environment. There were many reasons for this: it might be necessary to enable the participant to do her/his assigned task; the existing practice may have been inadequate or could be improved; or it might add to the analysis/synthesis in hand. I restrict myself, with one exception, to a few examples that incorporate a substantial amount of *workplace mathematics.* I only describe in detail those practices that are being discussed for the first time. Each case is classified as being one or more of the following;

1) replicating or adapting an existing practice or artefact;
2) introducing a known technique into a situation;
3) solving a problem or dilemma;
4) using a model and/or known facts and circumstances to reason with in order to extend an individual's or the team's knowledge; and
5) introducing of a new line of enquiry.

These categories were developed and refined using grounded theory.

11.2 Ramesh's checking of the NPV calculation for the Lyrebird vacant space provision

a) Calculating a discount rate The discount rate used by the Bank when calculating the vacant space provisions was 6% p.a. Quarterly cash flows were calculated and discounted. To check the NPV calculation Ramesh needed to determine the quarterly rate. He knew he needed to determine the fourth root of 1.06. This is an example of introducing a known technique – probably from his accounting studies, from a passing remark he made to me (2). He determined the fourth root using his calculator and he wrote the calculation of the quarterly rate on his working paper as follows: (1.0146738 – 1)*100 = 1.46738%. I noticed that Ramesh had some difficulty finding 'root' key on the calculator. Immediately after I completed the observation I asked him about this. He told me that he was not using his own calculator and he could not find the root button so he had used the power button instead to calculate $1.06^{1/4}$, as this was the same as calculating the 4^{th} root of 1.06. This is an example of using one's own knowledge to construct an original solution to a problem that arose (3); this is an example of transfer into a situation, rather than of transfer between situations.

b) Using the NPV function in the Excel programme Ramesh set up an Excel spreadsheet to calculate the NPV of the cash flows. He listed the quarterly cash flow totals on a spreadsheet and used an Excel function to calculate the NPV. As he worked he told me that one did not apply the NPV function to the first quarterly cash flow as the Excel function would discount it (which in the circumstances under consideration would have been incorrect). He therefore added the total of the first quarter's cash flow to the total produced by the NPV calculation. This is an example of replication of a known practice (1).

11.3 Techniques introduced by Eric into his work

a) Designing and using the Excel proforma to summarize the Bank's loans to particular customers – section 7.3 Setting up the Excel proforma was an example of reflective practice. In designing the proforma, Eric not only standardized the layout of the summary of the Bank's loans to clients, he locked into the proforma features that capture results of key calculations and comparisons. Designing the proforma is an example of inventing a solution to a perceived problem (3), i.e., creating a tool to facilitate repetitive work. Using the proforma in practice involved replicating and adapting existing practices (1).

b) Setting up the table to show that loans to the Possum group were mostly to Possum Networks – section 7.5 Eric decided that he only needed to consider the loans to Possum Networks as the loans to other group companies were insignificant. He followed a ubiquitous practice of accountancy and presented information justifying that decision in a table. Furthermore to save effort he adapted one of the tables in the Bank's papers. This is an example of adapting an existing artefact to suit the facts and circumstances under consideration (1).

c) Creating the cash flow for Possum – section 8.4 As the Possum Network accounts did not contain a cash flow, Eric decided to construct one. Eric from both his studies and his work on the Bank audit had substantial knowledge of the standard types of cash flows that could be generated from accounts. He used this knowledge to construct a cash flow to show how Possum Networks' operating cash flows were used. This practice involved using a known technique to solve a problem (2 & 3).

11.4 Cliff's practices on review

a) Widening the perspective taken of the market valuation of the Kookaburra debt by bringing the sale of the equity notes into the foreground – paragraph 2, subsection 8.2.5 c) This is an example of using the facts and circumstances of a particular case and the results from a model to shed new light on the model results (3).

b) Suggesting the use of ratios to express the relationship between debt and cash flows – paragraph 2, subsection 8.4.4 Cliff introduced Eric to a new and more appropriate analytical technique (5).

c) Using the order to sales ratio to help evaluate Wallaby's future trading position When Cliff carried out his final review of the Wallaby files, he read the directors' update on the trading position following the implementation of the Wallaby reconstruction. He noticed that that the quarterly orders to sales ratio for two quarters were reported to be greater than one (i.e., orders were greater than sales) and that the ratio for the second quarter was higher than that for the first. He cited this as evidence of improved trading in his commentary in the Wallaby working papers file. This an example of introducing a new technique to reason about a particular situation (2 & 4).

11.5 Tom's practices in review meetings

a) Reviewing summary schedules of Bank's lending to obtain an overview of movements in loan book during year At the beginning of Tom's first loan loss review meeting, he decided to get an overview of the loan portfolio by reviewing three schedules which summarized the make up of the loan portfolio and the bad debt provisions for the larger exposures. The review took the form of a line by line analytical review which highlighted the quantum of the changes that had taken place during the period under review. This enabled Tom and presumably Cliff and Eric who listened as he talked to build up their knowledge of how the portfolio had changed in the recent past. This is an example introducing a new line of enquiry (5) which also involved using standard analytical review techniques (1).

b) Introducing a new line of enquiry – testing the validity of using the Kookaburra debt valuation as the basis for calculating the bad debt provision – paragraph 5, subsection 8.2.5 d) and section 8.3 In Tom's meeting with Cliff and Eric about the Kookaburra provision, Tom introduced into the discussion consideration of whether or not the operational cash flows justified the then current market valuation of the debt (5). When Tom examined the Kookaburra cash flows during his review of Eric's files, he created an argument which disproved his expectation of a likely 50% write off of Kookaburra debt if it were to reconstruct and provided some justification for a 25% write off. During this process Tom used several accounting techniques and many mathematical relationships and models. Tom did not follow any standard method but used his expertise to integrate accounting and modelling techniques and known facts to evaluate the position (2 & 4).

c) Demonstrating how to use the Possum cash flow to reason about the recoverability of the Bank's debt – paragraph 5, subsection 8.4.4 This was an example of using the results of a model (Eric's cash flow) and known facts and circumstances to reason with and to bring new perspectives to the foreground.(4)

11.6 Reflective practice of other participants

a) Deciding on the date for a secretaries' meeting – subsection 5.2.5 Leonora carried out a rigorous logical process to determine the most suitable date for the secretaries' meeting. She used a routine which she had develop and improved over time. (Source: Leonora's comments during her post observation interview.) This is an example of importing a technique, which had been developed and refined over time by the participant, to solve a problem (2 & 3).

b) The spreadsheet style adopted by Sasha – table 2, subsection 5.2.4 Sasha, as a newly qualified accountant, joined the firm from a small accounting firm. When she joined she was surprised that the firm did not have rules regarding the way information and data should be set out in spreadsheets. She, therefore, decided to use those that she had used in her old firm. This is an example of replicating practice (1) and, unlike all the practices discussed, arguably one where the *workplace mathematics* is a marginal distinguishing feature.

11.7 Reflective practice and innovation; resolving perceived dilemmas and problems

When participants got stuck, did they stop and identify similar/analogous problems that they had dealt with in the past and seek to apply techniques that had worked in those cases resulting in a transfer of learning between situations/problems? Maybe. Maybe not. Asking these types of questions shift the focus from the active agent, the participant, and what s/he was doing to that of a differing perspective, the nature of the task in hand. The problem/dilemma is the participant's. Thus the following line of enquiry seems more suitable; 'How did participants progress tasks effectively when they were not sure how to go on?' This question focuses on what the participant imported into the work situation as opposed to what was transferred from one situation to another. It focuses on reflective practice and the act of innovation. This perspective is more in keeping with the situated cognition paradigm. When the question is so reformulated it is the narrow and wider contexts of work that provide most of the answers and the issue of transfer becomes subsidiary.

Above I have listed a number of situations in which participants reflected and introduced new ideas and techniques into a task in hand. Innovation seems to occur in two situations:

- The participant perceived a problem/dilemma in the course of carrying out a task and to resolve that problem/dilemma s/he introduced a new practice into the task in hand to take it forward. By introducing a new practice, I mean using a practice that either is not in progress or in use in the immediate work environment. This usually involved the participant using their own knowledge of a well theorized accounting or auditing practice or a practice that had become personal to the participant. This covers Ramesh's, Eric's, Leonora's and Sasha's work, apart from Ramesh's calculation of the discount rate.
- The participant introduced a new line of enquiry for itself or to rectify/ improve existing work. This covers much of Cliff's and Tom's innovation. It

is to be noted that all incidents referred to occurred as they reviewed audit work.

The examples listed above do not include a participant researching or exploring possible different approaches. I am not sure that this fairly represents the practices observed. For example, Joan, a New Zealand qualified accountant, prior to a meeting with a member of the firm's technical department researched the UK accounting treatment of purchased goodwill and Holly discussed informally with the client, a partner and me (on separate occasions) how an option in a particular contract might be valued for US accounting purposes. (I have not used the latter observation notes for any other purpose as Holly was so cross about the situation she could not stop talking about it and her frustrations, so invalidating the observation as evidence.) Dilemmas were generated out of particular tasks and the surrounding facts and circumstances. They were generally resolved as participants introduced and used professional/work practices or frameworks derived from those practices. The practices chosen were appropriate to the situation and were ones of which the participant had both knowledge and some practical experience. Dilemmas were not resolved by using a generalised higher level cognitive skill as such but by using well theorized models or practices or well practised skills. The exception was where Ramesh used the power function key on the calculator when he could not find the root key on the borrowed calculator – this was a case of transfer into a situation.

Like Lave's supermarket shoppers, the dilemmas and problems that arose, or were identified, as the participants worked were generally resolved satisfactorily. Individual effort based upon an individual's knowledge and experience was important. Most of the solutions were derived locally from contexts in the immediate work environment and/or the participant's or the team's own knowledge and skills. It is, however, to be noted that the participants themselves gathered together the resources and the information that they proposed to use as they carried out the task in hand. Sometimes problems were solved by gathering additional information. Otherwise where the immediately available resources did not provide the means to solve a dilemma/problem, participants, either on their own or with team members, used their own knowledge and experience to generate a solution, the solutions being based upon the participant's knowledge and past experience and practice, and what was thought to be needed. These solutions though innovative to the situation tended to be conventional to the workplace observed. Thus innovation was mostly determined by the task in hand, the resources which were readily available, the individual knowledge and skills of team members, appropriate work practices, and procedures of the firm and the team.

The issue of how cognitive skills are transferred from one task to another is far from clear. Several studies show that transfer is patchy (Ceci, Rosenblum and DeBruyn, 1999). I would like to suggest an alternative approach to the question of how learning and expertise is routinely introduced into a problem situation. Instead of considering whether a generalised higher level cognitive skill is or is not used in appropriate circumstances, it may be more appropriate to approach the issue with a different question. In carrying out a task how does an active agent resolve perceived dilemmas? My observations in this chapter suggest that this may lead to a more productive approach.

12 Continuity across dissimilar situations: the contribution of the individual and the team

12.1 Introduction

In this chapter, I first highlight some similarities and differences between mathematical practices observed, using examples and findings from this study, *Cognition in Practice, Street mathematics and school mathematics* and *Cognition in the Wild.* I then turn to consider social skills and *workplace mathematics,* individual intellectual effort and its contribution to a task in hand and so to the importance of what one knows and can do.

12.2 Comparison of findings across studies

12.2.1 Introduction

The everyday and work activities reported in these three books cover a wide range of activities. The experimental methods and objectives also varied and as I am using the results to show that there are similarities between the mathematical practices reported and my participants' practices, I first briefly discuss each study. In the following subsections 12.2.3 to 12.2.5, I summarize some of the findings reported and use them to compare and contrast with some of my findings. In subsection 12.2.6, I summarize emphasising the commonalities of the *workplace mathematics* observed.

12.2.2 Differing methodologies and objectives

Lave and *Cognition in Practice* (1988)

Lave set out to show that learning transfer does not account for continuity in activity (i.e., math practice) across settings and to develop a theory that could account for the continuities and discontinuities, namely that of situational specificity. She set up a psychological experiment that showed discontinuities in practice and performance between a math test (average per test: 70% (p.56)) and in the math used when shopping in a supermarket (almost no errors) and in a 'best buy' simulation test developed using examples similar to those observed on supermarket shopping trips (average per test: 93% (p.56)). The results from the 'best buy' simulation were also significantly better than the results obtained by Capon and Kuhn (1979 and 1982) in their 'best buy' experiments (only 44% of 150 participants solved both problems (Lave, 1988, p.102)). Participant observation together with contemporaneously taped commentaries (of the participants) and discussions (steered by the observer) enabled collection of data regarding math practice in the supermarket. Observers played a more active role in collecting data on math practice in the supermarket than I did as I sat beside my participants. However, the role of the observer was not unlike that of a reviewer in an audit meeting, although deliberately less helpful (p.160-164); this may account for much of the similarity in practices observed.

Participant observation was the main source of data for the weight watchers' math practice, though, in the course of their food preparation activities, participants were asked to prepare particular dishes. Interview techniques were used to understand family money management – not discussed in this chapter.

Nunes et al. and *Street mathematics and school mathematics* (1993)

Nunes et al. report upon half a dozen studies which demonstrate the differences between school mathematics and *street mathematics* and investigate the ability to solve problems similar or related to those that occur in the workplaces under investigation. The studies, amongst other things, look at the mathematical practices of children who are street traders, small farmers and students, carpenters and apprentices, foremen in the building trade and students, and members of a fishing community. Data about the work activities were gathered using interviews and ethnographic methods. Tests were administered to participants, with participants being asked to talk about what they were doing as they worked. The tests were based upon math examples observed *in situ* or discussed in the interviews. A major difference between these studies and my study was that the studies were set up to test particular hypotheses while the object of my study was primarily to find out what was done within the workplace. Again the situations in which the explanations were given bear similarities to discussions in review meetings.

Hutchins and *Cognition in the Wild* (1995)

Hutchins studied navigation on board US naval vessels. Navigation involves computing and plotting position at sea and providing the helmsman with information so that the vessel could be steered. It involves much mathematical computation, most of which in the circumstances under observation was locked into the various instruments and artefacts that assisted the navigators, though the navigation teams used some math as they worked. Hutchins' study used ethnographic methods and findings were based upon extensive observations and recordings. He used his observations to describe how cognition relating to navigation was distributed across time, in instruments and artefacts, through the organisation of processes and the team, and across and between individuals acting. I have used this to underpin my analysis of the contexts in which my participants worked. A key difference between my participants and Hutchins' was that Hutchins' generally followed standard procedures as they piloted vessels, while my participants had considerable freedom as to how they carried out tasks in hand. Hutchins reports one exceptional incident which occurred when the *Palau's* engines failed as it was being piloted into its home port; this is discussed at length in subsection 12.2.5 as it is of particular significance from a *workplace mathematics* viewpoint.

12.2.3 Everyday activities and *workplace mathematics*

Lave (1988) in *Cognition in Practice* reports:

> "In best-buy problems in the supermarket, problem-solving processes often involved "left to right" calculation (decomposition of a number into hundreds, tens, and ones starting with the largest and working through to the smallest), recomposition, rounding, ratio comparison, transformation of both problems and solutions in the course of problem solving, use of the environment as a calculating device, ..." (p.58)

There were twenty five supermarket shoppers in the AMP study. Nearly all calculation in the supermarket was mental/oral. The shoppers made few errors. However problems in the supermarket were occasionally abandoned in the middle of a calculation for an alternative solution to the problem, e.g., postponing a purchase or selecting a larger item; practices that would count as errors in experimental terms. Calculations were abandoned when the numbers were not easy to manipulate, e.g., in this extract from discussion between a shopper and her daughter about buying sauces:

"Daughter: 18
Shopper: 18 ounces for 89, and this is? ...
Daughter: One pound, seven ounces –
Shopper: 23 ounces for a dollar 17. [Speaks ironically.] That's when I whip out my calculator and see which is the better buy" (p.166).

If this comparison had arisen in the course of one of my participants' work that is precisely what s/he would have done! Also decimal/fraction conversions were surprisingly frequent in the everyday activities of AMP participants (p.69).

Shoppers used mental/oral math almost exclusively as they shopped. My participants wrote, interpreted and discussed texts incorporating substantial financial information. When writing or working alone their preferred methods of calculation were electronic for almost all but the simplest of calculations so, amongst other things, solving the problem of intractable numbers. Where the integrity of the calculations was deemed important they checked both for completeness and accuracy. Lave's participants effectively checked their calculations by performing a series of related calculations as they reasoned during particular shopping episodes. But there were marked similarities between the participants' practices. They transformed problems, e.g., Eric decided to look only at Possum Networks once he had established that the Bank's loans to other members of the group were insignificant. My participants abandoned tasks but not out of personal convenience, e.g., the Honeyeater loan loss review was abandoned because the Bank sold its loans, and Tom decided that Eric should not review the operations of Kookaburra as they were too many uncertainties attaching to the underlying business going forward. In conversation my participants' mathematics practices bore similarities to the shoppers; they rounded numbers and used estimates, articulated assembled calculations in a left to right format rounding to two/three significant figures and set up many comparisons and occasionally used ratios. Eric's written commentaries on the loans had similar characteristics.

The relationship between the shopper and the supermarket had a major bearing on the shopper's activities. In particular, individuals were in control of their activities within the supermarket, both generating and controlling the problem-solving processes. Problems might appear to be 'out there' within the setting but it was the shopper's option to 'see' them. The shoppers owned the problems. In the course of solving problems/dilemmas, shoppers continually transformed them.

Shoppers assembled quantitative relations (created calculations) in various forms, including price and quantity comparisons. These occurred at the end of a largely qualitative decision-making process when a shopper faced a dilemma. If arithmetic was used to solve the dilemma, it was employed when the number of choices was not greater than three and rarely greater than two at a point where the shoppers have no strong qualitative preferences (Murtaugh, 1985). The arithmetic problem solving enabled decision making. Lave comments that the use of arithmetic by shoppers as a common medium for problem solving is itself "an interesting problem." She suggests that the way in which arithmetic is used to justify shoppers choices is as a symbolically powerful image of rationality, utility and objectivity since, in the supermarket, calculation may be the most immediate means of asserting the rationality of choice once the qualitative criteria of choice have been exhausted (p.157-158).

My participants were generally not in control of the larger problems with which they had to deal. They were usually given tasks to do. However they were substantially in control of the problem-solving processes and through the implementation of a task acquired ownership of the problem. Where ownership was not acquired, e.g., during Ed's initial reviews of Wallaby's results, the tasks were less well done. For all my participants, other than Leonora, *workplace mathematics* had a major impact in determining or shaping tasks as many of the tasks involved assessing the reasonableness of financial results as representations of situations and events. Mathematics was not only used to solve problems, it was also used to present and organise information for a subsequent reader. My participants, like the AMP participants, used arithmetic both as warrant and rhetoric for choices made.

Lave provides illustrations of how shoppers and weight watchers used the environment to enact math practices. During a shopping trip, a shopper noticed a packet of cheese seemed overpriced. To satisfy himself on this point, he found a similarly sized piece of cheese and compared the price tags. In another situation, a weight watcher was asked to use three quarters of two thirds of a cup of cottage cheese in a meal. He did not calculate $^3/_4$* $^2/_3$. He filled two thirds of a cup with cottage cheese emptied it onto a chopping board and patted into a circular shape, quartered the shape with a knife and took three quarters; in other words he enacted the calculation. These two examples like the referencing, filing and finding particular working papers and the revaluing of the Kookaburra loan involved using math/*workplace mathematics* to implement action in the world.

12.2.4 The characteristics of *street mathematics*

Nunes et al. report (1993, p.55) that, in the oral arithmetic used by street and other traders, meanings were retained as calculations are set up and performed and the arithmetic relations and the situation were part of the same schema (equivalent of idealised cognitive model). Consequently solutions to problems are constructed to reflect the practicalities and the mathematical representation of the situation, e.g., counting out change. They consider, in particular, that the closeness of the representation of the situation and the arithmetic in the schemas developed in street mathematics (which includes different types of workplaces) simultaneously preserves meaning and allows for greater flexibility in the routes to a solution. The preservation of meaning also allows answers to be checked for sense. Similar practices were observed with my participants' oral and written practices.

The oral arithmetic practices of my participants (and Lave's shoppers) were similar to those in Nunes et al.'s studies in that: after rounding numbers to two or three significant figures, they worked from left to right in hundreds, tens and units or in tens and units to one decimal place; and they preserved both the relative value of numbers and their commercial meanings as they worked through a problem. Also there was flexibility in oral practices used (Cliff understanding the Kookaburra valuation). My participants, unlike Nunes et al.'s (see below), did not fail when numbers became intractable because they used electronic means of calculation.

Perhaps the main distinguishing feature of my participants was their success with written calculation practices. This is partly because Nunes et al. mostly generated written practices in unfamiliar test situations. However, my participants were successful because they used electronic means of calculation extensively and they preserved relative values and situational meaning as written calculations were composed and recorded, and also because they made situational sense was made of answers. Disciplines to aid completeness and accuracy also contributed to success. One could say that many of the disciplines of oral arithmetic workplace practices have been carried over into the accounting and other work practices observed.

In many of the studies carried out by Nunes et al. the performances of 'experts' were compared to that of students or apprentices and sometimes their arithmetic was tested in school-like tests. A general finding is that the groups of 'experts' were better at solving the problems based upon workplace practices than either their comparator groups or in tests of arithmetic. Another general finding is that as numbers became intractable more problems were solved incorrectly or were unsolved. Perhaps the most important general finding is that 'experts' learnt/developed and used in their workplaces a range of problem solving techniques to solve problems generated therein and related problems elsewhere[1]. For example, the building foremen when solving proportion problems mainly tested specific hypotheses or found and used the scale relationships, when the relationships were not immediately obvious. They did not use the general rule of three algorithm. Students mainly found and used the scale relationship; they did not test hypotheses and only one used the rule of three for two of the four problems tested. In some of the specific examples of problem solving reported in *Street mathematics and school mathematics* iterative methods were used (p.61 from Grando (1988) and p.95).

Like Nunes et al.'s, my participants progressed their tasks in hand mostly using the facts and circumstances of the case (the particular) together with practices and procedures developed within the workplace – some were institutionalised and some were personal. They generally modelled the particular using well theorized models of accounting; iterative model building techniques were commonplace. There were differences in competence between expert and apprenticeship performance, e.g., the contrast between Eric's and Tom's performances in constructing and using a cash flow model. It is perhaps worth noting that in my study such variation in performance was demonstrated by Eric alone – his expert creation of the loan summaries and his slightly hesitant construction of the Possum cash flow. Tom, like the building foremen, tested hypotheses; when he estimated the likely write off of the Kookaburra debt, he tested two hypotheses rather than solving his equation which represented the problem under consideration[2].

[1] As is shown by the performance in experimental situations.
[2] See subsection 8.3.4 b), paragraph 3 for another example.

12.2.5 Navigating a helicopter carrier

Hutchins (1995) reports that on a routine voyage most of the math practice other than that of taking readings of bearings and soundings, recording them and plotting the position on the chart was hidden from view. To plot a position triangle on a chart three bearings are required. The bearings are read off the scale in the alidade, transmitted by telephone to the bearing recorder who in turn records the bearing in a log and may repeat out loud. The plotter who works beside the bearing recorder hears the bearing or reads/checks it in the log, sets his hoey and marks a line (known as a line of position (LOP)) on the chart starting from last plotted position. Depth soundings are also taken and recorded. Coming into port bearings were taken frequently – the intervals varying between one and six minutes. Performance is far from error free. Pelorus operators may have difficulty in finding the landmark from which the bearing is to be taken. Bearings may be misread, misheard, and recorded incorrectly. Error repair often is a team effort, e.g., correction from memory or the bearing record, or from an understanding of where the vessel is in relation to the landmarks and where it is marked on the chart.

The type of activity in which my participants engaged was substantially different to that of the navigators on the *Palau.* It is therefore not surprising that the *workplace mathematics* was quite different. Perhaps the most significant difference was that on the *Palau* most of the key mathematical practices were locked into the navigation instruments and were therefore unproblematic, while my participants invented mathematical models and practices using well theorized frameworks, both to represent situations and events and to interpret and evaluate them. The tasks in hand, the time frame over which they were performed, the settings in which the activities took place and the roles played by members of the two different teams were all quite different. So in reflecting upon similarities, one has to exercise caution. The practices of navigation and auditing/administration were mostly learnt through practice on the job, a relatively formal apprenticeship system and through observation and some coaching via the on the job learning. Each member of the team was given specific tasks and when team members interacted with each other they effectively checked each others' performance by reaching consensus with respect to what were perceived as critical facts and there was some joint repair of errors.

I now turn to the incident on board the *Palau* when its engines failed as it was being piloted into its home port so the plotter, who was the quartermaster of the watch, and the bearing recorder had to compute the bearings to be plotted on the chart manually. To calculate the true bearing of a landmark the compass bearing needs adjusting as follows:

True bearing = compass bearing + deviation + variation + relative bearing.

$$T = C + D + V + RB.$$

Between the engines failing and the carrier reaching its berth, 66 lines of position (LOP) (3 are required to fix each position on the chart) were computed. This episode is a significant reported incident of learning and adaptation of *workplace mathematics* under pressure. Here I highlight some key changes that occurred in performance during the plotting of the first 41LOPs:

- **Methods of calculation** For the first 12 LOPs, the plotter calculated C+RB+V mentally, from time to time using scales on the hoey (nautical protractor) and paper and pencil as aids to calculation. The structure for the computation was driven by the availability of the data. The recorder assisted with some of the calculations. The plotter complained that he could not calculate fast enough, missing a couple of plots. The bearing recorder got up from the place where they were working and got a calculator. Thereafter the calculations were performed on the calculator; this reduced considerably the load on working memory, thus releasing time for other activities. It is to be noted that errors were made and corrected in the course of keying numbers into the calculator.

- **Algorithm used** For the first 24 LOPs the plotter and recorder used the algorithm, C+RB+V. Deviation (D) was not included; this did not matter until LOP 22 because it had been at or near zero. But before compass bearing 22 was taken, the *Palau* swung round causing the triangle fixing position to increase in size. The plotter realised what was wrong and walked over to the helm station and read the deviation card posted on the compass stand. Thereafter the algorithm, C+RB+V+D, was used. The order in which the readings were added varied from plot to plot, until a normative structure of the computation emerged around plot LOP 33; it took the form of (C+D+V)+RB. This follows the form of the mnemonic well known to navigators.
- **Impact of changes in environment and procedures on work** The carrier's anchor was dropped just before LOP 5 was plotted. After plotting LOP 12 the plotter instructed the bearing recorder that position fixes should be changed from every minute to every six minutes and about the same time the recorder introduced the calculator. The change of position of the carrier just before LOP 22 was taken enabled the complete true bearing algorithm to be activated.
- **Teamwork** Throughout the plotter and recorder collaborated in their work. Hutchins reports that the most important consequence of the introduction of the calculator was it became a medium through which the plotter gave the recorder instruction in calculating the true bearing algorithm and thereafter the plotter and recorder shared the burden of calculating the true bearing, with the recorder increasingly taking more of the burden. Between plots LOP 25-33, a stable method of calculating the true bearing emerged; this involved the recorder gradually accepting and calculating in accordance with the plotter's concept of the algorithm which had developed during the course of the crisis.

Despite the complete difference in task and setting there are many interesting parallels in task performance between my participants and the two navigators. The plotter brought pre-existing knowledge of the task and its enactment to the situation, i.e., calculation of the true bearing (cf. Tom's structural schema for reviewing loan loss provisions). The full algorithm was only enacted when the partial calculation was seen not to be working in practice (cf. Eric's construction of the Possum Networks cash flow). The calculator was used to ease the burden of calculation. The setting was used to enable performance, e.g., the scale on the hoey to aid calculation (cf. adapting the table of the loans to Possum Networks in the Bank's review) and responding to the existence of the inappropriate size of the triangle fixing position on the Mercator chart (cf. Joan's response to the trading losses produced in her draft of the goodwill model). Throughout the crisis the plotter and recorder talked to each other and worked in close collaboration; in particular they shared the calculation burden including repairing errors (cf. interchange between Cliff and Eric as Cliff learnt about the Kookaburra valuation model). The plotter used the calculator to show/instruct the recorder in how to calculate the true bearing (cf. Cliff's feedback to Eric on the Kookaburra review and Tom's coaching of Eric re analytical reviews). Normative performance arises with practice and repetition (cf. Eric's preparation of the loan loss summaries) and correct practice is developed iteratively in response to observed results (cf. Joan's model building techniques).

12.2.6 Concluding observations

As is outlined in this section there are both significant differences and similarities between arithmetic practices in everyday life and differing non-specialist workplaces. My participants' **written *workplace mathematics* practices** often demonstrated the same degree of competence as Lave's and Nunes et al.'s participants' oral math practices. Common features of competent performance were;

- accepting ownership of designated tasks;
- being able to transform problems to take account of available resources and information;
- reducing the intractability of numbers by rounding to an appropriate degree; and
- preserving in written texts the relative values and situational meaning of numbers as written calculations were composed and situational sense is made of results including those arising at intermediate stages.

My participants also:

- overcame the intractability of numbers by using electronic means to perform almost all calculations;
- actively collaborated with team members through both informal (asking for help on a minor point) and formal means (the review system);
- had or made to time to understand the task in hand and surrounding facts and circumstances; and
- were experts in auditing/administration and related *workplace mathematics* techniques or were guided by experts.

Hutchins' participants in the emergency described above demonstrated similar competences.

There were similarities in conversation between the calculation practices of my participants and Lave's and Nunes et al.'s. All assembled calculations. Also problems were generated and resolved in settings with solutions being substantially structured by past experience and cues in the immediate environment. Lave's shoppers and my participants used quantitative information as warrant for statements, abandoned problems but for quite different reasons, and occasionally enacted a mathematical activity.

For Lave's and Nunes et al.'s and my participants, heuristic techniques were mainly used to solve/resolve problems/dilemmas and they were mostly learnt and reproduced *in situ.* Repetition of similar tasks/subtasks in everyday activities and within specific workplaces generates competence/expertise in performance (cf the extended example above and much of Eric's work).

For all participants, the rules of arithmetic (as opposed to the practices used) embedded in tasks were, with the exception of rounding practices, the same.

In this conclusion I have briefly emphasised some of the key similarities observed across a limited number of different work and everyday practices. This should not disguise the substantial difference between the activities themselves, their settings and the knowledge and skills the participants brought to the activities. It is, however, the similarities that have significance for mathematics education and functional mathematics and for this reason they have been emphasised.

12.3 The individual participant's contribution

12.3.1 Introduction

Analysis using both grounded techniques and Hutchins' distributed cognition framework highlighted the importance of participants' own knowledge and skills and cognitive effort. The similarities between *workplace mathematics* practices across dissimilar tasks/activities described above suggested it would be productive to explore the nature of the cognitive effort made and to consider the nature and extent of individual knowledge and skills used.

Thus I reanalysed an incident, namely Cliff's initial review of the Eric's working paper on the Kookaburra loan, to bring these issues to the fore. I used my observation notes of the review from section 8.2 (Eric's working paper (exhibit 3), the Bank's valuation (exhibit 4) and my notes and the tapes of Eric's meeting with Cliff (exhibits 5 and 6)). I reanalysed the data several times to illustrate different aspects of the work observed. I used the data:

1) to show what Eric and Cliff did from an auditing perspective;
2) through deconstruction to isolate some of the *workplace mathematics* observed;
3) to show how the results of *workplace mathematics* was used from purely a social perspective;
4) to construct charts of the contexts and settings;
5) through deconstruction to highlight the cognitive effort made and its distribution; and finally
6) to foreground individuals' extant knowledge and skills.

The results of 1) and 2) are not reported in any detail as the original analysis (section 8.2.5) generated substantially the same results.

12.3.2 What Eric and Cliff did from an auditing perspective

By the end of the discussion Cliff knows about the Bank's provisioning policy and that it has a £48.5m provision against its loans to Kookaburra and that the carrying value of the loans is approximately £9m higher than their market value. As he learns about the situation, he is not happy with the size of the provision and asks for Eric's view. Eric starts to justify the status quo – the stance he has gone along with in his draft paper – but he then articulates his doubts which seem to have grown as he and Cliff worked on the Bank's valuation schedule together.

This meeting achieves much more than its stated purpose – briefing Cliff. Three things happen in this meeting: Cliff understands the position; Cliff ensures that he and Eric have a common understanding of that position; and Eric revises his view as to the adequacy of the provision.

This is an analysis of the auditing. It is the work within which *workplace mathematics* is embedded.

12.3.3 Some of the *workplace mathematics* observed

A substantial amount of *workplace mathematics* was performed as can be seen from this summary:

- reading and understanding calculations;
- rounding and estimating practices;
- describing a mathematical model and its usage;
- constructing a logical mathematical argument;
- demonstrating publicly the mathematical structure of the debt valuation model;
- using the model to derive a result;
- demonstrating how the result was derived using mathematical statements;
- understanding how the result is derived and confirming agreement publicly;
- using the agreed result to evaluate the situation – being interpretation of a result this might not be *workplace mathematics*; and
- reading the result from previous period's valuation enabling an unspoken comparison to be made.

12.3.4 How the results of *workplace mathematics* were used from purely a social perspective

Eric describes, and he and Cliff understand and use the Bank valuation of the Kookaburra debt, a mathematical model, to assist them assess the Kookaburra bad debt provision but much more was done with *workplace mathematics,* when one admits Luhmann's view that codes are conduits of meaning. Mathematics is, amongst other things, used as such in financial reporting.

Eric and Cliff's discussion is substantially **focused** around the numbers in the Bank's debt valuations.

When Eric summarises the overall position (subsection 8.2.5. a)), he uses numbers and mathematics to:

- **organise and reduce complexity; and**
- **focus on key points.**

Much of the information given to Cliff is mathematical. To **aid understanding and the successful completion** of a subtask commercial information and meanings and mathematical structures are interwoven. This is similar to the practice of the street traders and other workers in Nunes et al.'s studies.

Understanding the Bank valuation schedule (subsection 8.2.5 a), exhibit 5) When Cliff asks about the current position, Eric **turns to the valuation schedule** (subsection 8.2.3, exhibit 4) and **starts guiding Cliff** through it. As Cliff seeks to understand the Bank's valuation schedule, he **articulates his temporary difficulties in a**

mathematical way as he seeks help from Eric – "Which way round is that? Is it an under or over provision?" He **is also highlighting what he needs to sort out** to his own satisfaction. Initially Eric guides Cliff as he, Cliff, seeks to understand the model but then **Cliff takes over control.** When Eric appears to falls behind Cliff in re-understanding the model, Cliff **uses calculation both to focus Eric's attention and to structure his understanding** so they can come to an agreed understanding. Together they work through the model to **achieve common understandings** of both how the model works and the result it shows, both of which **are publicly acknowledged.**

Refocusing the discussion Cliff **refocuses the discussion** by seeking confirmation of his understanding of the Bank's position with reference to the Bank's provisioning policy expressed **by reference to the market value of the debt.** Cliff later **asks Eric if** "we are happy with [the status quo]?" (subsection 8.2.5 b), exhibit 6). Initially Eric is inclined to say yes but the agreed meaning derived from the model **leads Eric to revise his opinion.** In setting up the comparison with the previous period's results, Cliff **gathers further evidence of a mathematical nature** regarding the current provision.

Here the results of *workplace mathematics* are being used in discussion to organise the understanding and evaluation of a situation. In particular, Eric and Cliff use the results to organise and reduce complexity, to focus and progress discussions and to progress the task in hand. They are used both in communication and to guide action. Eric and Cliff probably did further *workplace mathematics* as they used the results of the *workplace mathematics.*

12.3.5 Charts of contexts and settings

The following four charts provide a diagrammatic picture of the context, setting and the larger environment in which the task was realized. All the *workplace mathematics* is hidden from view here because the analysis is too coarse grained.

The context here is the Kookaburra loan loss review. The task in hand is Eric's briefing of Cliff. Eric's draft working paper and the Bank's valuation schedule were the physical resources in the immediate setting. Eric's knowledge derived from writing the working paper plus Eric's and Cliff's professional knowledge and experience were the key drivers of the cognitive input and resulting action. Understanding and evaluating the bad debt provision was a collaborative effort with the work being shared between Eric and Cliff. In the background support for these efforts was provided, amongst other things, by:

- the audit plan and the organisation of the audit team;
- professional and firm practices and procedures including the rules for assessing bad debt provisions; and
- the technological infrastructure.

Table 1 Structuring environment – analysed mostly from a business perspective

Context - structuring event - assessing Kookaburra loan loss provision
Task - Eric briefs Cliff prior to his preliminary review
Realisation of subtask was divided into four subtasks: Eric briefed Cliff; Eric and Cliff worked together to understand how model worked and what it implied; Cliff asked Eric if he was happy with the Bank's provision; and Cliff set up comparison with prior period.

Setting and the structuring resources in the immediate environment			
Eric and Cliff working together in a meeting			
Eric*	Eric's working paper	Bank's valuation model	Cliff*
*including the ability to read and interpret texts, to decide what to do, to use arithmetic and *workplace mathematics,* and to reason in discussion			

Reference resources immediately available to Eric and Cliff		
Standard accounting models*	Working papers files	Some Bank papers
Firm, team and individual practices*		Common professional practices*
known to Eric and Cliff		
*in this case, these were part of Eric's and Cliff's own intellectual resources		

Environment - underpinning organisation		
Team	Organisation of teamwork	Bank audit plan
Firm's procedures and practices		Bank's procedures and records
Technological infrastructure ----- generic and firm/Bank specific		

These charts surface two key questions. How did Eric and Cliff know what to do and do what they did? They had to rely on their own intellectual resources. Thought, cognitive effort, preceded action. Both relied upon knowledge and skills of everyday practice, such as the 3Rs, professional and work practice, and the ability to get around and work in an office environment competently.

I now turn to consider the cognitive effort Eric and Cliff made as they sought to understand and assess the bad debt position relating to Kookaburra.

12.3.6 Cognitive effort and its distribution

Theorizing about cognition was (and for some possibly still is) quite problematic for those working within situated cognition frameworks, as such frameworks place emphasis on what is observed *in situ.* Lave in *Cognition in Practice* suggested that cognition is stretched across the person acting, the setting and the arena but did not really explain how that happened. Hutchins tackled these problems directly. He provides insights into individuals' thought processes as they work as part of a team through what they did and what they said to one another. He also shows how the results of past cognitive efforts were locked into tools and artefacts and how those results were accessed at and in work. He thus argues, in my view successfully, that cognition was distributed both across time and between individuals. Subtasks, all of which required cognitive effort, were not just shared out between individuals; some were accomplished only through collaborative effort where the cognitive effort was shared.

I now focus upon Eric's and Cliff's cognitive effort.

In the analysis below, I use a distributed cognition framework to locate that effort. Column 3 of table 2 provides a chronological account of what was done in the meeting. Column 2 describes Eric's work on Kookaburra prior to the meeting and column 1 highlights other events and extant knowledge that were clearly relevant to the assessment of the provision in the meeting. I also categorise the cognitive effort made into high, medium or low. The nature of task/subtask, the time spent on it and the degree of concentration manifest in the observed behaviour all contribute to my assessment of the effort made.

Table 2 – Distributing cognitive effort – assessing the Kookaburra provision

Time →

Past → ***Present***

Prior events/learning	Eric's work on audit	At the briefing meeting
Eric designed the proforma. He had used it many times.	Eric constructed table. Effort high (I observed preparation of other tables.) Table constructed because Bank papers not in suitable format for audit team's work.	1) Eric and Cliff read summary table. Effort low Because all work done.
Eric knows about • bad debt provisioning in banks and • about Bank's business, policies and systems. (Also Cliff.)	Eric (and Cliff) had meetings with client where Bank's position explained. Effort high I observed intense concentration at two client meetings.	2) Eric sets scene using table and other knowledge. Summary succinct and efficient. High cognitive content delivered with low effort because work done before meeting.
Cliff knows about Kookaburra because he has dealt with it at other financial institutions.		3) Cliff understands as he looks and listens attentively. Effort low
Cliff is an experienced reviewer of credit positions.		4) Cliff refocuses to assessment of provision. Effort Low
Bank valuation model presents information but not particularly clearly. Cliff and to a lesser extent Eric are experienced at understanding and using valuation models.	Eric decided to use the model. He annotated it to include comparison of carrying value against market value. Using model saves effort (No effort assessment as activity not observed.)	5) Eric finds his annotated version of the Bank's valuation schedule. Effort low

Time

Past → ***Present***

Prior events/learning	**Eric's work on audit**	**At the briefing meeting**
• Teamwork in action. • Eric and Cliff work together to come to a publicly expressed common understanding. • There is sharing of cognitive effort.		6) Cliff and Eric understand and use model. 6 a) Eric begins to guide Cliff through model. Eric – effort low Saves some effort for Cliff 6 b) As Cliff begins to understand he takes over and makes sense of model. Effort – high 6 c) Cliff asks for and obtains Eric's confirmation of his understanding. Cliff – effort medium Eric – effort high
Cliff and to a lesser extent Eric know how to assess bad debt provisions.		7) Cliff refocuses discussion to assessment of debt provision. Effort low
	Eric wrote working paper and created audit file. Effort high Eric's assessment of provision is not sufficiently reflective.	8) Cliff asks Eric for his view of provision. Effort low Eric revises opinion from OK to ? Effort high
Bank valuation for prior period in audit working papers file.		9) Cliff sets up comparison with prior period. Effort low Eric and Cliff make comparison. Effort low

Footnote: Cognitive effort made categorized into high, medium or low. My assessment took account of the nature of task/subtask, the time spent on it and the degree of concentration manifest in the observed behaviour.

The cognitive effort highlighted is that of Eric and Cliff. It is substantial and consists of deciding what to do, listening and understanding, reasoning and the intellectual effort in doing (some of which itself is reasoning).The table illustrates several other important points:

- The macro and micro organisation of the audit and the task in hand helps structure what is done and when it is done.
- At each stage as the task is progressed, Eric and/or Cliff decide what to do or listen and seek to understand.
- Cognitive effort is distributed over time, subtasks and team members, enabling competence in performance and the expertise that resides in the team to be brought to the task in hand.
- Knowledge and experience of auditing loan loss provisions both structures and facilitates execution of the task.
- Knowledge and skills in arithmetic, modelling and logic both enable and are part of the auditing practices observed.

Furthermore the cognitive work necessary in the meeting was reduced as a result of:

- all Eric's preparatory work;
- the use of the Bank's valuation; and
- Cliff's and Eric's extant knowledge of the Bank's affairs.

Consequently Cliff and Eric were able in the meeting to focus on issues they deemed to be critical.

Column 1 of the table highlights some of Eric's and Cliff's prior knowledge and extant skills that contribute to task performance. This brings us to another aspect that is essential to enabling Eric's and Cliff's competence – extant knowledge and skills.

12.3.7 Foregrounding individuals' extant knowledge and skills

Eric and Cliff brought two distinct types of knowledge and skills to the assessment of the Kookaburra bad debt provision – those relating to the practice of accounting and auditing, and everyday skills that enable them to be articulate, numerate, and literate. Wittgenstein's epistemological views are particularly apposite here. Through training (including formal and informal), apprenticeship and practice, Eric and Cliff had acquired the necessary knowledge and the skills (including facility of technique) in the two types of practice, namely that which made them articulate, numerate and literate and that which enabled their work as accountants. Eric's and Cliff's competent performance of new and unfamiliar tasks or of tasks involving substantial new information was predicated upon both learnt knowledge and competence/expertise in both types of practice with the latter being in part dependent upon the former. They, for example:

- organised and used resources;
- read for understanding;
- were able to communicate effectively within the context of a review meeting – both as an initiator and recipient;
- formulated, understood and answered questions;
- used arithmetic and logic;
- understood, used and created simple mathematical models;
- used accounting and auditing models and practices; and
- made auditing judgements.

The execution of the work required deliberate effort. Some was executed without thinking too much about it. Using Wittgenstein's terminology, Eric and Cliff used bedrock practices. Some performance was for one reason or another effortful. What Eric and Cliff already knew and could do, including being able to use cues and references in working papers, substantially determined the nature and competence of performance. Thus prior education and training is of critical importance.

A further gain from these insights is that human agency is a major contributor to continuities of practice within and across practices. Continuities in practice will tend to occur in the same or similar circumstances where usage is sanctioned through education, training and practice. However as a result of human agency continuities may occur when knowledge and skills derived from education, training and practice deemed appropriate to particular circumstances are enacted in new differing circumstances, provided that the circumstances in which the agent is working do not proscribe the innovation. The new usage may or may not harden into practice.

12.4 Concluding remarks

In this chapter, I show my findings have some external validity. My findings are situationally specific to particular situations/events which occur within two business units of the UK assurance division of a large international accounting firm. In earlier chapters, I have highlighted some similarities across participants, tasks and situations observed. In the first half of this chapter I show that there are also important similarities between some of my findings with respect to *workplace mathematics* practices and some of those reported in *Cognition in Practice, Street mathematics and school mathematics* and *Cognition in the Wild.* Given the dissimilarities between the participants' activities and the settings in which they take place, it does not seem unreasonable to make some tentative general conclusions about the nature of everyday *workplace mathematics* practices.

In the second part of the chapter I reanalysed a small incident, Cliff's initial briefing about the Kookaburra loan loss review, to inquire into the relationship between the participants, the task in hand, setting and other contexts. In this reanalysis, I have sought to describe and organise the structure of task, setting and environment in a series of diagrams – table 1. This brings a number of number of points to the fore. First it provides a framework for viewing the wider and narrower contexts. Second by its absence from the table, it highlights the embedded nature of the *workplace mathematics* observed. Finally and most importantly this change of emphasis in the latter part of the reanalysis away from the task in hand and the embedded *workplace mathematics* to context highlights three critical aspects of individual agency observed:

- skilled social usage of the results of *workplace mathematics* – section 12.3.4;
- the centrality of the cognitive effort of individuals acting alone and collaboratively to the achievement of goals – section 12.3.6; and
- the critical importance of individuals' extant knowledge, skills and experience – section 12.3.7

It is open to an individual to use their extant knowledge, skills and experience to assist with carrying out a task in hand, provided that the individual is permitted to exercise agency beyond carrying out the task in a prescribed manner. Given the continuities (and discontinuities) observed in and across practices in similar and dissimilar settings, I suggest that individuals' extant knowledge, skills and experience exercised alone or in conjunction with others partially accounts for similarities in *workplace mathematics* practice over time and across situations/events in different settings. This suggestion does not detract in any way from the importance of the influence of normative practices, available resources within the task setting or other situational factors but it is a key observation if this study is to be of relevance to the development of the functional mathematics curriculum. This brings me to the point where I can re-categorize and summarize my findings.

13 *Workplace mathematics* observed: contributions to knowledge

13.1 Introduction

This qualitative study provides insights into how employees do/use mathematics in their everyday work. It sits within the situated/distributed cognition frameworks. Two purposes underlie the research; to find out how mathematics is actually performed in the course of carrying out everyday work and to provide data and analysis to inform the current debate relating to functional mathematics, i.e., its what and how and the what and how of its teaching and learning. My findings (for detail refer to chapters 5 to 12) and conclusions present a complex picture as I have sought to keep both the mathematical practices used and the contexts in which they were used in the foreground.

The *workplace mathematics* observed was tightly woven into the work practices observed. To provide insight into *workplace mathematics* and the embedded mathematical practices, my findings in the conclusion are separated into a number of strands. First, I discuss contributions to and from theory, and define and explain what *workplace mathematics* is and in the process reflect briefly upon its relationship with mathematics of the discipline of mathematics. Second I deconstruct my detailed findings to itemise:

- the mathematics used;
- the purposes for which *workplace mathematics* was used;
- the nature of the performance observed; and
- learning and innovation observed.

Given the complexity of the situations observed, these findings overlap. I then return to consider briefly the importance and influence of context and settings upon the performance observed and the agency of individuals. Finally in the next chapter I turn to consider very briefly the implications for mathematics education in secondary schools and for possible future research.

13.2 Contributions from and to theory

Before I re-synthesize my findings, I consider the impact of theory on my observations and analysis. In this study I chose to use well developed theoretical concepts to underpin my research to enable focus on my research questions and, incidentally, to avoid re-inventing the wheel. My study is a qualitative study within the situated cognition paradigm: in particular, Hutchins' distributed cognition framework provided me with an effective way of analysing everyday work in an institutional setting such that I was able to uncover the *workplace mathematics* of my participants as they engaged in their everyday work. Like Lave and Hutchins, I sought to keep cognition out of the 'head' and where in the limited cases I had to speculate about my participants' thought processes I mainly used George Lakoff's concept of Idealised Cognitive Models. My epistemology was derived from some of Wittgenstein's remarks upon how we come to do and to know. I used Luhmann's systems theory in several different ways. His systems theory at its most general is a highly theoretical framework which enables analysis of sociological questions. I used it both to justify the selection of observations I and my participants made as we sought to make sense of complex data and to distinguish between thought and communication. I also used it to create a distinction between the mathematics of the discipline of mathematics and *workplace mathematics.* Furthermore I adopted Luhmann's idea that codes are used to simplify complexity and to enable communication, and that mathematics is used as such.

In the case of Luhmann's systems theory, the contribution of this thesis is unusually, and perhaps uniquely, to apply aspects of his sociological framework and analytical tools to the field of mathematics education.

My qualitative study is an empirical study and my findings are empirical. I have not sought to, nor have I, advanced the theories themselves. However, my methods and findings contribute new insights into the uses to which the theories can be put.

Most work in the situated cognition paradigm emphases findings in particular contexts and discontinuities between practices. In contrast I have used Hutchins' theory of distributed cognition to enable a focus on both continuities within a practice and their realisation in similar but nevertheless different tasks in similar but different contexts. Furthermore, the comparison of my findings with the studies discussed in chapter 12 suggest that it may be possible to identify a significant number of findings about the mathematics used in everyday life work that are of general relevance, in effect widening the relevance of certain findings derived from local studies in the situated cognition paradigm.

My epistemology of knowledge is derived from Wittgenstein's remarks on how we come to do and to know through enculturation/induction into a practice through practice combined with guided teaching and learning as to the rules of the practice. My study shows how Wittgenstein's epistemological viewpoint was realised within the professional practice of auditors and ordinary office practices within an assurance division of an accounting firm and thus throws light upon how we come to do and to know.

13.3 *Workplace mathematics*

What is *workplace mathematics?* For both Lave and Nunes et al. the question of what is mathematics was not problematic. For Lave it was arithmetic and for Nunes et al. it was elementary mathematics, which is realised differently in school and in the street/ workplace. In the context of business could it be – using mathematical models where modelling is the process by which aspects of possible/actual 'real world' situations/events are represented mathematically? None of these concepts were adequate for my purposes as detailed analysis of the work observed showed the mathematical activity incorporated into work, *workplace mathematics,* to be more complicated.

I approached the question from both theoretical and practical viewpoints. Consideration was given to what is meant by mathematics and how that relates to the mathematical activity incorporated in the work practices observed. I use Luhmann's systems theory in general and in particular his concept of the differentiation of the social system to distinguish the mathematics of the discipline of mathematics (a subsystem of the social system) and the mathematics that is now used outside that system. As a result of the differentiation of the social system, they can and do influence each other[1] but are generally realised and developed independently of each other.

My data supports this key distinction. My participants did not regard themselves as doing 'mathematics' (i.e., of the discipline) with the possible exception of where they were specifically checking the arithmetic integrity of numbers in accounts; they saw themselves as engaged in tasks that were enabling an audit or office administration. Occasionally, my participants appeared to use mathematical results or methods borrowed directly from the discipline of mathematics to resolve or illuminate problems, e.g., when Tom modelled the loan repayment period as an algebraic equation or Anil used fractional indices to calculate a fourth root. They applied knowledge/skill learnt elsewhere to the task in hand. It was not possible from the data collected to identify where the knowledge or skill was acquired. However, it is possible using Luhmann's concept of structural coupling to theorize about the transfer mechanism. According to Luhmann there is no operative overlap between communication and the individual consciousness. Language in the form of speech and written texts couples these systems (for more detail see section 2.3.2, particularly the third paragraph).Thus in the two cases quoted above, an individual consciousness which had acquired knowledge/skill in one or more settings through participation in communication through language transferred that knowledge/skill into communication in another setting. Of course, such transfers occur as a matter of routine within a given work practice including embedded *workplace mathematics.*

Using the theoretical distinction between the mathematics used in the discipline of mathematics and that used outside, I set out to find mathematics in use in the workplace. This led me to develop the definition of *workplace mathematics* grounded in the mathematical activity in the work practices observed in phase I (chapters 5 and 6).

Workplace mathematics *is mathematical activity carried on within the workplace as part of ordinary work practices. Two defining principles must be present for a mathematical activity to be recognised as workplace mathematics;*

- *part or all of the work practice (system) must incorporate at least one mathematical concept*[2] *which is used in accordance with the rules of mathematics; and*
- *usage occurs when a participant acts in a way that is consistent with the mathematical rules that apply to the concept/s such that either meanings are given to information, or actions and/or events in the 'real' world are facilitated.*

The definition's theoretical antecedent is Nunes et al.'s definition of *street mathematics.*

The concept of *workplace mathematics* is grounded in the work observed and is conceived of as mathematical activity which contributes to and facilitates a task in hand. The activity of workplace mathematics comprises more than just the application of mathematics and mathematical techniques to work in hand; it comprises carrying

[1] In Luhmann's systems theory partly through the process defined as structural coupling.
[2] See section 6.2 for definition.

out part (or occasionally all) of a task. For example, the *workplace mathematics* embedded in Tom's determination of the write off of the debt which Kookaburra might have had to make if it had to reconstruct was an integral part of the whole task in hand (section 8.3); it cannot be separated from the whole. It was also the case with Eric's construction of the Possum cash flow (section 8.4). Sometimes only a small amount of *workplace mathematics* was incorporated into a task, e.g., the discussion of whether or not the secondary market prices for Kookaburra debt properly reflected the then current value of the debt. The mathematical activity observed in use was not only used to represent 'real world' events/situations, it formed part of some procedures followed and was used directly to structure learning about some events/situations. *Workplace mathematics,* as defined, covers all these activities.

Perhaps a simple example best illustrates the nature of *workplace mathematics.* As part of checking the veracity of the directors' emoluments note in T's financial accounts, Gary sought to check the figure used for the finance director's salary to T's underlying records. Gary reconciled (proved) the figure in the remuneration schedule to a copy of the record of the salaries paid direct to directors and employees by deducting the personal pension contributions paid by T to an insurance company on behalf of the finance director. He then wrote up his findings in the client working papers using a formal reconciliation (a mathematical proof). It is possible to distinguish the mathematics and the mathematical techniques used, the *workplace mathematics* and the task in hand. Mathematics and mathematical techniques were embedded in the task. First to get from the figure in the remuneration schedule, Gary **deducted** the personal pension contributions paid **using a calculator** and **agreed** the answer to the figure shown in the record of the salaries paid. He then **wrote up the calculation** on the remuneration schedule that formed part of his working papers, describing the meaning of each number in the calculation. **A columnar presentation** was used for the calculation. The two activities described above are the *workplace mathematics* performed and these activities formed part of the task showing that the appropriate figure for the finance director's salary was used in the note to the accounts.

Such analysis is valid as one searches for mathematics in use in ordinary work. However from Gary's and the audit team's perspective, Gary was simply auditing a situation i.e., verifying the information in the directors' remuneration note. In the process of carrying out the task he used *workplace mathematics.* I observed nothing in his behaviour to suggest that he was adopting practices other than standard auditing practice, i.e., checking the information and then recording his findings using a standard referenced reconciliation. Although in writing up his findings as a reconciliation he almost certainly would have said that he was using some mathematics if he had been asked.

This generates another pertinent question. By grounding the definition of *workplace mathematics* in the work practices observed, can *workplace mathematics* be distinguished from the work observed? Or is what was observed just part of the practice of accountancy/auditing or administration? Or is it legitimate to argue that mathematics in use can be distinguished in those practices? My description, deconstruction and analysis of the practices observed, as set out in detail in chapters 5 and 7 to 11, show that mathematics was used extensively by my participants as they worked and that generally the mathematics was embedded in the task in hand.

Consequently it is legitimate to distinguish elements of the activity as *workplace mathematics.*

However, following such a route is not without dangers. The Cockcroft report (1982) sought to identify the mathematical knowledge and techniques used across a wide range of occupations. In the process it decontextualized the mathematics and mathematical skills from the work in hand and aligned them to the school curriculum and thus failed to identify what was actually done and how the mathematics was actually used. In my description and analysis I have sought to avoid this pitfall by keeping contexts in view as I seek out and analyse the mathematics in use. This process has produced rich findings showing both how individuals do mathematics as they work and the wide range of uses to which mathematics is put, albeit that the observations were made within a small part of one business.

My participants' actions were substantially determined by the tasks in hand and surrounding contexts. Other than Leonora, the administrative assistant, they used *workplace mathematics* extensively. Leonora used some. Extensive usage was found partly because of the work chosen for observation and partly because the telling episodes chosen for analysis were episodes which were likely to be, and were, rich in usage. Much flowed from the contexts. However, as I have shown far from all was determined by the context (see section 12.3 in particular); participants both individually and collectively brought much knowledge and many skills to the tasks in hand.

I now turn to the knowledge, skills and learning observed, and afterwards briefly reconnect the practices with the tasks in hand and the settings in which the tasks were accomplished (section 13.7). As explained above *workplace mathematics* was part of/embedded in the work practice observed. It was not a separate distinct activity. However it involves the application of mathematical concepts. Below I deconstruct the work practices observed separating out the mathematics and the mathematical and techniques used (section 13.4), and the usage to which the mathematics was put (section 13.5). This analysis is of relevance to members of education and training communities.

13.4 The mathematics in *workplace mathematics* categorized

The mathematics used is discussed under four headings:
- calculation;
- presentation of data in numerical form;
- logic and proof; and
- mathematical modelling.

Most of the practices referred to are taken from the summaries, including the interim conclusions, in chapters 7 to 11. Here they are reclassified into the above categories.

13.4.1 Calculation

My participants carried out numerous calculations. The calculations made and techniques used varied depending partly upon the nature of the work undertaken. There were differences between written and oral practices. There were also differences between practices adopted when an individual sought to understand, write or review texts. Here the methods of calculation observed are discussed under two sets of headings:

- the first being written practices, oral practices, and exploratory calculations; and
- the second being rounding and estimating practices, errors and error repair, and participant competence.

Written practices

Written practices here refer to creating texts and work done alone when reviewing working papers files. Participants checked calculations by punching them into calculators, e.g., Holly during the review of the draft accounts and Gary checking the table of the directors' remuneration. They set up/wrote calculations as they created/revised text by punching them into a calculator (the Possum table and the notes below the Excel spreadsheet of the Possum cash flow) or typing them into an Excel spreadsheet (Eric into the Wallaby summary table and Joan as she revised the goodwill calculation). In the case of the Excel spreadsheets, the calculation was set up either using arithmetic, spreadsheet algebra or the Excel function keys. Except for the simplest calculations, calculations were performed electronically – a key finding. Calculations which were considered to be important were checked for accuracy and to ensure completeness and accuracy of input, e.g., Eric as he created the Wallaby summary and Holly as she reviewed the draft accounts. Data capture in the electronic form of the Excel spreadsheet aided these processes.

Calculations used in tables were set up carefully with the table being completed to the participant's satisfaction. Calculations were used in a slightly different way in narrative texts. Sometimes they were copied direct from source texts and sometimes they were punched into calculators as texts were being written, with the answers being inserted into the text. Sometimes there was sufficient information in the resulting text to enable the calculation to be re-performed, sometimes reference to source texts was necessary and occasionally I could not reconstruct the calculation, e.g., paragraph after exhibit 4, section 10.7.

In the work practices observed, including those of the administrative assistant, the setting up and checking of calculations were an essential part of the participants' *workplace mathematics* practices. The contexts surrounding the calculations substantially governed the nature and the contents of those calculations but the actual setting up of the calculations themselves depended substantially on the participants' own knowledge and skills and was a key part of the work done.

Oral practices

My participants calculated as they articulated their thoughts in discussion. The calculation practices observed during discussions differ substantially from those used as texts were created. First and foremost they were usually performed mentally, although Tom did use the calculator for the calculation of the interest on his estimate of the long term Kookaburra debt, possibly because the numbers, including the place value implications, were not obviously malleable. During most of the discussions observed, participants used or referred to written texts, including continuing to look at those texts as they performed the calculations. Calculations were read out of the texts and numbers from the texts were used to set up calculations, though sometimes rough estimates were used. Numbers were rounded to two or three significant figures when identifying quantities and to enable easy of manipulation in calculations. Calculations were articulated before they were performed; they were set up using a left to right format, i.e., starting with the largest unit. (The data collected and analysed was insufficiently detailed to be able to comment upon how the mental calculations were actually performed.) Meanings, including units, were kept attached to numbers as calculations were set up. Furthermore performance was sometimes a collaborative effort and sometimes the structure of a calculation was demonstrated by pointing to numbers in the texts and articulating the relationship between the numbers (discussion between Eric and Cliff relating to the valuation of the Kookaburra debt).

Exploratory calculations – for the detail see section 10.3

An exploratory calculation is a calculation made by a participant to aid understanding of a text or a situation. Exploratory calculations, which by their nature are *workplace mathematics,* are made to aid sense making, e.g., they may be used to structure thinking as a participant struggles to make sense of a situation (Gary and Ramesh) or they may be used to generate new results that provide further insights into the meaning of particular situations (Tom). Throughout my fieldwork, I observed participants using this type of calculation as they strove to understand particular facts and circumstances. Certain distinctive characteristics seem to attach to them. They are performed on a calculator. As a participant tries to understand something, s/he reaches for a calculator, which has often been deliberately laid on her/his desk so it is readily available, and rapidly punches a calculation into the calculator and looks at the answer. If the answer is satisfactory, s/he may move on, write an explanatory note in the working papers or indicate that it is so by saying something like "We're there". If the answer is not satisfactory, the participant may carry out further quick calculations, decide on another way to resolve the problem or dilemma or even abandon a line of thought. Other distinctive features are: the calculations are performed more rapidly than those made to check the accuracy of workings; initially they tend to be done for oneself and only shared if successful; and also they tend to be done when a participant is working alone on an aspect of a task. Explanatory calculations are powerful flexible tools that aid learning and sense making.

Estimation and rounding practices

As my participants worked they made extensive use of estimation and rounding practices. These differed depending upon whether they were creating/writing texts or discussing texts. When Eric created tables and his Excel cash flow model in his working papers, he converted all the loans to sterling and rounded the numbers to the nearest £'000s/£ms and the roundings were performed correctly in accordance with the rules of arithmetic. Sasha and Joan did not use rounding practices when they created their Excel spreadsheets. Numbers in narrative texts were quantified and tended to be rounded to two or three significant figures, including Sasha's spreadsheets. Particular care was taken to round correctly where the texts formed part of a formal record or a public document.

In discussions and in review meetings, estimates tended to be much cruder. Numbers tended to be rounded to two rather than three significant figures. The rules for rounding numbers up and down were often applied in a cavalier fashion with numbers not being rounded to the nearest significant figure. Participants occasionally only estimated the answers to calculations set up in conversation. Numbers and calculations played a key role in discussions relating to the understanding and interpretation of records of events and situations. I suggest that the rounding and estimating practices observed enabled participants to concentrate on the commercial implications, their key focus.

Errors and error repair

My participants' work was not error free but they made relatively few errors and most were corrected. The following 'persistent'[1] calculation errors were made by participants in the episodes chosen for analysis:

- Eric initially included a credit balance in a sum of debit balances for the Wallaby table – he corrected this input error almost immediately;
- Eric initially added an increase in debtors in the Possum cash flow; he adjusted this logic error when he reflected on his initial calculation;
- Joan initially incorrectly calculated the cost of additional sales in the goodwill valuation. She realised it was wrong in the course of generating the figures for the revised model. She then set up a better calculation; and
- Eric in his paper of the reanalysis of the Wallaby results described creditors as increasing, not decreasing as was the case. This error was not corrected. It was probably an error of transcription as the reduction had been used correctly in the immediately preceding calculation.

Two errors were made and corrected as the Koala summary table was constructed. Errors tended to be eliminated from participants' work as they made sense of the calculations created.

Participant competence

Participants were very competent in setting up and performing calculations. They were skilled and knowledgeable users of arithmetic. Generally they chose to perform an appropriate calculation and set up that calculation without too much thought or effort. They made few mistakes when calculating and these were almost always picked up and corrected, either almost immediately or during the review process. For written work electronic means of calculation were used for all but the simplest calculations and calculations were checked for completeness and accuracy where they were deemed important. Mental calculations were used in oral discussions observed: these were generated from information in written texts, set up, described and performed in conversation. Also for critical calculations time was spent ensuring that there was a common understanding of and agreement as to what was to be calculated, the result and/or its meaning. Generally for all calculation participants kept commercial meanings attached to the input and output numbers as they calculated. They or other team members were interested in the answers produced and their commercial meanings; routinely they made sense of the calculations. All these practices taken together explain the participants' general high level of competence.

[1] By persistent I am excluding errors which were made as numbers (and possibly calculations) were first input to a calculator or a formula cell as my field notes are not sufficiently detailed and only appear to record exceptional incidents.

These practices are to be contrasted with practices at Key Stage 3 and 4 where the emphasis is often only on the correct performance of the calculation itself.

Reading texts silently – limited evidence for practices used

Participants read many papers containing calculations and results from mathematical models. I did not collect any direct evidence of the calculation methods they used when they read the papers silently as I did not ask them to articulate their thoughts. Given the speed with which they read, and evidence from discussions and actions that immediately following the reading of a piece of text, the participants seemed to accept most of the calculations read in the texts as being correct. Sometimes when the meanings attaching to calculations were not clear or the participants wished to extract/create new meanings, they performed exploratory calculations, this being most obvious when they used calculators to perform those calculations.

13.4.2 Presentation of data in numerical form

Participants presented and summarized financial/numerical data in tables and commentaries. They also used similar texts as they worked. I distinguish this and the related practices from modelling and model usage, although some data display could be considered to be modelling in that the data displayed represents situations/events. The distinction I am emphasising here is that the information is displayed for itself and in the process is usually used to organise and simplify complexity. To create the tables and commentaries participants used information from other texts, including computer listings and tables prepared by others, and information gathered from discussions, tables, trend analyses, and financial commentaries. Below I discuss a number of mathematical practices which were used to display data under the following headings: classifying balances, tables, comparatives, ratios, and statistics and trend analysis.

Classifying balances

Participants classified and subtotalled account balances to organize, simplify and summarize information for their and others' use. The classifications adopted were those of accounting and banking practices. Some were quite rigid, e.g., Sasha divided programme accounts into stock (programmes paid for but not yet transmitted) and accruals (programmes transmitted but not yet paid for), and Eric excluded credit balances from his loan summaries. Other were less so, e.g., the loan classes used in the summaries prepared by Eric followed the Bank's classification except where consolidation or sub-division might aid the audit presentation (use of other as a classification and the sub-division of the bonding lines by reference to the collateral in the Wallaby summary) and Tom's subdivision of Kookaburra's creditor balances. All participants worked with information that had already been classified.

Tables

Data was often displayed in two way tables with a row/column format. My participants created and used many tables (for detailed discussion see chapter 7). Texts of some mathematical models kept for the record, e.g., the Kookaburra valuation and the Possum cash flow, were often preserved in table format and used incidentally to present information.

- Tables were used to organise and present information in numerical format (Sasha, Leonora and Eric).
- They were also often used to simplify and summarize data.
- Care was taken to ensure tables prepared for the record were accurate and highlighted the numbers/results deemed important.
- The structures embedded in the tables enabled the itemisation and subtotalling and totalling of key numbers.
- They also locked in a number of key mathematical relationships, e.g., units, comparisons of totals (Sasha), confirmation that loans outstanding category by category were less than approved facilities, and the working papers version of the Kookaburra valuation not only calculated the market values of the junior debt, it was modified to display the total carrying value of the loans beside the total of the market values.

The usage of tables is discussed further in section 7.6.

Making comparisons

Comparatives in the sense of one number being compared with another were the most commonly used analytical technique, partly because the previous comparable period's results are set against the current period's results routinely in financial accounts. They were used to both present information in numerical format and to make and enable mathematical inferences, i.e., a figure was larger or smaller than, or the same as, another. My participants created and used this type of comparative extensively. Eric's commentaries and analyses of company results contain numerous comparatives from financial accounts. Tom at the start of his review of the Bank's loan portfolio discussed the Bank's schedules showing the totals of each category of lending on a line by line basis by comparing them with the totals for the previous comparable period. Eric's table of loans outstanding at time of the loan loss review compared the amounts outstanding with approved limits on a line by line basis. Sasha, when she was reviewing the television company's stock recording systems, set up tables to show that the value of films shown but not yet billed was the same for each department – here comparisons were made to check that different systems produced the same results. Eric, Gary and Ramesh checked numbers in one set of schedules (source documents) to schedules generated from the source documents to ensure that the input to accounts, tables or calculations was both accurate and complete. Joan set up a comparison of the results produced by her goodwill valuation with the price paid by BFJ for the goodwill.

Comparatives were the only form of trend analysis that I observed my participants creating and using extensively.

Ratios

Eric and Cliff created, used and reported a few numerical ratios prior to interpreting results and Ramesh calculated unit measures as he sought to understand the vacant property provisions. Tom showed Eric how to use ratios when he taught Eric how to do analytical reviews of accounts for the purpose of assessing bad debt provisions (appendix 3). Ratios were used analytically to enable interpretation of events/situations.

Statistics and trend analysis

My participants did not use statistics extensively. Percentages were calculated and used. Trend and other statistical analyses in client papers were read and were used a little to understand and evaluate situations, e.g., when Eric wrote about Wallaby's forecasts (subsection 9.2.4).

My participants created no graphs, and hardly created or used trend analysis and statistical measures, as I observed them. They created and used these tools less than I had expected. This is partially, but not I believe wholly, explained by the activities actually observed. By way of contrast, when I talked to the management of a building society and reviewed its board papers to establish the mathematics used by its employees for a paper prepared for Adrian Smith's review of secondary school mathematics (2004), I found that many of the society's employees made extensive use of graphs and trend analysis in the management of its day to day activities.

Arithmetic and statistics (for data display and comparatives) were not the only mathematical methods used. Logic and mathematical modelling were used extensively.

13.4.3 Logic and proof

Logic was used when results were checked and interpreted, in mathematical modelling and as audit trails of numbers were created and checked (for their arithmetic integrity). Logic was used in mathematical arguments for a wide variety of purposes. Tom during his review of the Kookaburra debt provision created a logical argument showing that the results from the underlying operations justified the secondary market prices and hence the validity of their use to evaluate the bad debt provision [1][1]. Leonora in deciding the best date for a meeting executed a logical process from which she determined the best date; the process used was a personal routine that she had developed over time [2]. Participants used reconciliations (accounting technical term for numerical proofs) to prove/explain how one number is reconciled to (or derived from) another [3]. Gary reconciled the managing director's salary shown in a salaries listing to a remuneration schedule that supported the figures used in the annual accounts. Eric reconciled the cash generated in the Possum Networks business to its usage when he constructed the Possum cash flow; the reconciliation enabled him to satisfy himself that he had both adequately quantified the cash generated by the business and itemised its usage. Audit conclusions were often derived using logical inference which involved quantities [4]. Holly deduced that intercompany debt was used to finance the payment of an intercompany dividend. Logic is an essential tool of mathematics and all the examples quoted involved deriving conclusions from mathematical representations.

Tom used two other mathematical techniques as he assessed Kookaburra's financial position. He set up an algebraic equation that represented the debt that Kookaburra

[1] Numbers indicate the types of situations in which logical arguments were used.

could sustain and he tested three hypotheses of his expectations. It is also to be noted that Nunes et al. (1993) report that experienced participants tested hypotheses to solve problems similar to those that they met at work.

13.4.4 Modelling

My participants' mathematical modelling and model usage is described and analysed in chapter 8. The creation and use of mathematical models involved extensive use of *workplace mathematics.* The detail is not repeated here. The distinctive features of model creation and usage are summarized in detail in section 8.6. The use, including the creation, of mathematical models pervaded many of my participants' work practices. The models ranged from simple pricing and valuation models to fairly substantial cash flow and goodwill models. The extent of model usage is powerfully illustrated by the extensive use of smaller models nested within larger models – Tom's cash flow analysis of Kookaburra (subsection 8.3.5, table 1) and Joan's goodwill model (subsection 8.5.4 d)). Underlying all was the model of the profit and loss account and the balance sheet.

Models were not only created for the record, they were created for immediate use – see Tom's creation and use of the Kookaburra cash flow model. Models were used extensively in conversation and discussions. Eric built/used a fragment of a model as he perceived that to be sufficient for the task in hand (subsection 9.2.3). Analysis of texts/records of models alone would have underestimated both usage and model creation. In particular, it would have failed to provide a full picture of usage as records represent only a trace of what is done and they do not necessarily show what was significant for the user.

My participants' modelling practices had several distinctive features, many of which do not resonate with the framework of mathematical modelling that appears in many textbooks. The term that best describes the approach to the modelling observed is heuristic. My participants only created a model when there was no suitable model that they could appropriate or adapt; then they used well theorized models of accountancy and auditing as frameworks, rather than creating models from scratch – a important use of distributed cognition. With one exception (Eric's loan summary proforma) they did not build the model and then populate the model with the facts/assumptions relating to the case under consideration; they modelled particular facts and circumstances. Models were created through a iterative trial and error building process through which the particular facts and circumstances of a case were modelled progressively – the processes are both described and analysed in detail in chapter 8. My participants did not move from a world of commercial meanings to a world of mathematical modelling and back again, they operated in both worlds simultaneously with the world of commercial meaning having precedence. As participants created and used mathematical models, they kept the commercial meanings of the numbers closely attached to numbers in the models both as they built and/or used them in reasoning. As already mentioned, I consider that this practice enabled mathematical competence, sense making and reflected the fact that the mathematics was mostly a means to an end. A key exception observed to this practice occurred when significant glitches to the inputs, or logic of, the model arose; then the focus centred on getting the mathematics of the model right.

None of the models used by my participants were mathematically sophisticated. They were, however, generally fit for purpose when built and reviewed, and were used proficiently. They were also used in many different ways, some of which were surprising. In the next section I summarize briefly the different ways my participants used *workplace mathematics.*

13.5 Usage of *Workplace mathematics* observed

13.5.1 Usage

My participants used *workplace mathematics* extensively; usage was much more diverse than is suggested in some discussions of mathematical modelling (subsection 6.4.2). Usage is described and analysed in detail in chapters 5 and 7 to 12. Here I highlight significant features of the usage observed. But first I list and reference participants' main uses of *workplace mathematics*. These include:
Chapter 5

- Making logical deductions – Holly's use of accounts
- Reconciling two numbers – Gary's reconciliation of the managing director's salary
- Enacting a logical process – Leonora setting out secretaries' availability in a table and then using the table to make decision
  For more on logic and proof see section 13.4.3 above
- Checking calculations for completeness and accuracy – Holly, Gary and Eric
- Using the accounting firm's alpha numeric referencing system to locate papers and numbers and to file papers – all participants

Chapter 7

- Ordering information – Wallaby and Possum tables
- Summarizing and presenting information – Wallaby and Possum tables
- Simplifying complexity observed – Wallaby and Possum tables
  Also Possum cash flow
- Enabling and making comparatives – Wallaby table
  Sasha's construction of the programme accruals spreadsheet incorporated all these processes (subsection 5.2.4)

Chapter 8

- Creating/adapting and using mathematical models – the substantive subject matter of chapter 8
- Aiding decision making and/or making decisions – Kookaburra valuation model
- Implementing a decision – revaluation of Kookaburra debt in the Bank's books and records
- Supporting reasoning – cash flow analysis formed the framework for the construction of the logical argument to estimate the possible quantum of the Kookaburra write off
- Using to reason – Possum cash flow and models nested in larger models, e.g., the Kookaburra and Possum cash flows and Joan's goodwill valuation
- Structuring and guiding the collection of additional evidence – Joan's second draft of goodwill valuation

Chapter 9

- Writing authoritative texts – commentary on loan note repayments (subsection 9.2.3)
- Warranting statements – as above
- Providing rhetorical emphasis in written arguments – as above
- Using the fragment of a mathematical model in narrative text – as above
- Creating texts that others can use to make logical and mathematical inferences – commentary on future prospects (subsection 9.2.4)

Chapter 10

- Assisting in understanding financial information – Gary and Ramesh making sense of texts
- Using exploratory calculations to scaffold sense making – see subsection 13.4.1 above

Chapter 11

- Enabling a task – Ramesh's, Eric's and Leonora's innovation
- Extending and deepening discussions and written commentaries – much of Cliff's and Tom's innovation

Chapter 12, subsection 12.3.4

- Using the results of *workplace* mathematics to focus and direct enquires and discussions to aid understanding and evaluation of a situation

In section 12.2, I report in some depth on some important similarities between those of my participants and some of those reported in *Cognition in Practice* (Lave, 1988), *Street mathematics and school mathematics* (Nunes, 1993) and *Cognition in the Wild* (Hutchins, 1995).

13.5.2 Aspects of usage

There is complexity both in the *workplace mathematics* used by my participants and in the way in which they used it. Usage extended beyond that of the quantification and presentation of results expressed in financial terms, the creation and use of mathematical models to reason about events and possible/hypothetical situations and the use of financial information and results in reasoning and logical arguments. In making the case for other uses, it must not be lost sight of that my participants used *workplace mathematics* extensively for precisely these purposes. However it was also used for other purposes. It was used in action, e.g., to revalue debts, to file and find papers, and to compose a fragment of a logical mathematical argument using a series of extracts from other texts. It was used routinely in written texts comprising audit evidence. In particular, it was used to strengthen and warrant statements and to provide rhetorical emphasis in written arguments. *Workplace mathematics* was also used

- to aid the organisation of information in texts;
- sometimes to focus and guide the reading of texts and discussions; and
- to scaffold an individual's thought to enable learning and understanding.

The focus on numbers in texts was central to the review of some texts, particularly accounts and other models, and also to the choice and assimilation of much information from other texts. The usage of *workplace mathematics* thus extends beyond individual cognitive effort and the expression and recording of the results of that effort. It can be enacted, serve as evidence and be used to enable sense making or as a tool/signpost in discussions or as texts are read.

Consideration of an unsatisfactory text in chapter 9 not only brought particular uses to the foreground, it raised other issues. Eric copied and/or précised some text without any evidence that the choice of text involved the use of *workplace mathematics*. He also assembled text which enabled others to reason using *workplace mathematics*; again it was not clear that he himself used *workplace mathematics* as he created the commentary. Eric drew inferences from figures and results that cannot be wholly justified from a mathematical point of view and his analysis of the half year results was not adequate. This surfaces two issues with respect to *workplace mathematics*. Copying and précising financial information containing evidence of past use of *workplace mathematics* may not involve the use of *workplace mathematics;* Eric's usage is not *workplace mathematics* because he did not appear to use mathematics or the rules of mathematics as he selected the text for use. On the other hand Eric drawing the wrong inferences could still be *workplace mathematics* as it is probable he intended to use it in accordance with the rules of mathematics and logic. Using *workplace mathematics* to write the commentary on the half year results is *workplace mathematics* even though it was used in a way that was not fit for purpose.

Not all numerical activity is *workplace mathematics*. In section 12.3.4, I show how Eric and Cliff used the results of *workplace mathematics* to focus and progress their discussion of the Kookaburra loan review; much of the numerical activity highlighted almost certainly did not involve using *workplace mathematics*. Neither did Leonora's use of the numeric codes for the stationery order; the description generated by inserting the code to the programme drove Leonora's behaviour rather than the code itself. Nor was much of Sasha's numerical activity relating to the programme accrual and the discussions with Sonjoy – see paragraph starting **Numerical activity that is not mathematical** in subsection 5.2.4. Although the activities are not *workplace mathematics* as such, these activities show how the results of *workplace mathematics* are used to organise and simplify complexity and to aid communication.

13.5.3 Performing *workplace mathematics*

Much *workplace mathematics* is performed in and for the public domain; usage varies depending upon the nature of the intended audience and collaboration. Both add to the complexity of what is done. Some of the mathematics done was performed for self, notably exploratory calculations, that in notes in notebooks and rough working papers, and in reading and reviewing papers for understanding; and some was done for others, e.g., as presented in working papers and reports. This is not a surprising finding.

What however is a more interesting finding is the nature of usage as individuals work together collaboratively. Team members in discussion sometimes together set up, performed and used calculations/models; sometimes each member of the team did the calculation/used the model while at other times members of the team sequentially performed parts of the calculation or interpreted the model. In both cases conversational checks were often made to ensure that members had agreed common understandings. As the calculating/modelling proceeded conflicts of opinions/interpretations arose and were resolved so allowing new conclusions to emerge. Teams also often worked to establish common understandings and judgements of key numbers, calculations and results produced by models. *Workplace mathematics* and the usage of its results also played an important role in social interaction. On occasion, it acted as a pivot to focus and guide discussions and effort, provided techniques to reduce and organise complexity and was a means of expressing consensus as is demonstrated subsection 12.3.4. Sometimes records indicate that common understandings had been achieved but more often than not texts are silent as to the nature and extent of those understandings. There are analogous collaborative processes in which mathematical performance is shared between individuals within the iterative processes of creating and reviewing working papers.

Usage in working papers is also affected by the intended reader. The mathematics and mathematical models used in the creation of working papers are often initially created for the originator of the paper to satisfy him/herself on a point but the formalisation/finalisation of the records of the models and their results are produced primarily for others. Records of analysis of accounts numbers are created in working papers to be read and interpreted by members of the audit team (including the creator), subsequent teams and a few others. Results from mathematical models (often including sufficient documentation of the model) are set out in texts which form part of formal audit evidence. Mathematics and mathematical models are an integral part of detailed working papers. Working papers include a lot of superfluous information, so key information is generally highlighted or referenced for an intended reader.

Thus work in and for the public domain both structures and influences what is done. As tasks were generally in the end accomplished both satisfactorily and competently, it is necessary to consider some of the factors that enabled competent performance. Participants' skills and knowledge and context both influenced performance; and their influence is considered in the next two sections.

13.6 Observations about participants' skills, knowledge and learning

13.6.1 Competent performance

Although the mathematics that my participants used was not particularly sophisticated, it was often used with considerable sophistication. Much of what was done was governed by task in hand, the resources available and an ability to take advantage of work already done. But my participants' agency, knowledge and skills, experience and openness to learning were critical. Without them nothing would have been done competently.

13.6.2 Personal agency

My participants both required and showed considerable personal agency as they organised and undertook the work in hand. Once they had been given and/or decided to carry out a particular task, they had to make many choices as they carried out that task. This included finding out about and understanding events/situations and assessing the results of their work, either alone or together with other team members (chapters 5,7 and 8). To complete tasks successfully they had to rely upon their extant knowledge and skills and their experience, even where instructions, available resources and information substantially structured what was to be done. They also used feedback from discussions and the review process. They were open to learning and they learnt (chapter 10), and when necessary they innovated (chapter 11). This was as true for Leonora as it was for Gary, Eric, Joan or Tom. Work was not just a matter of following well practised routines. Furthermore my participants' extant knowledge and skills as realised in teamwork substantially determined and limited what was done.

Cognitive effort As my participants used *workplace mathematics* they engaged in considerable cognitive effort, either alone or collaboratively. Like Lave and Hutchins I have generally kept my observations and analysis 'out of the heads' of participants. However each action observed was either preceded or co-incident with thought/intellectual effort, though much of the load was borne by or incorporated in the technological and intellectual tools used. Reanalysis of a small incident, Cliff's initial briefing about the Kookaburra loan in sections 12.3.5 and 12.3.6 illustrates both the distribution and extent of that effort.

13.6.3 Knowledge and skills of *workplace mathematics* brought to a task

Generally my participants performed their work including the embedded *workplace mathematics* competently. My participants brought two distinct types of knowledge and skills to the tasks in hand – those relating to the practice of accounting and auditing, and everyday skills that enable them to be articulate, numerate and literate. Both incorporate mathematical practices. Participants were skilled users of arithmetic and spreadsheet algebra. They had good to excellent working knowledge of many generic mathematical models that are part of commercial and financial accounting. These skills and knowledge were applied to the tasks in hand as they were realised. In turn personal knowledge and skills were honed and developed through the tasks in hand.

Participants chose to use appropriate mathematical operations when calculating almost without exception – the main exception being Joan's attempts to calculate the cost of additional sales in the goodwill valuation. Participants could and did create models or parts of models without reference to other team members or other sources. This was, of course, only necessary where the immediately available resources did not contain appropriate examples or substantial cues. Participants used other analytical techniques that were mathematical in nature, e.g., comparatives, ratio analysis and analytical reviews of accounts, although in Eric's case sometimes with less confidence and competence (section 8.4.3 a) and section 10.7).

13.6.4 Practice and Experience

Practice and experience both had a bearing on what and how things were done. All my accountant participants were skilled in their use of arithmetic; given the extensive practice observed, it is reasonable to surmise that practice created/improved competence. The degree of expertise in model building depended in part upon practice (compare Eric's expertise in preparing the loan summary table with his hesitations when he built the Possum cash flow) and in part upon professional expertise (Tom's versus Eric's expertise with cash flow models). Eric was practised in writing factual commentaries on companies' financial results but did not know how to carry out an analytical review of accounts for a loan loss review until he was shown by Tom. Also when Gary embarked on auditing the remuneration schedule, a task he was doing for the first time, he took time to understand the schedule and to work out what he should do. He completed part of the task in hand successfully but only with considerable effort and resilience.

13.6.5 Time, perception and commitment

Knowledge, practice and experience are not enough to explain competent performance. Other factors were also important. These included taking time to do things (table 2, section 12.3.6), perception of the task in hand, commitment to results and openness to learning. Participants took time to do things properly. They took time to understand and define the task in hand. They took time to gather and understand information relating to a task. They took time to perform tasks and where the results were deemed important/significant to check them. Perception of what the task in hand was had a bearing on what was done. The most striking example of this is Eric's and Tom's differing perceptions of what Eric's role was in the loan loss review. Eric considered that his primary purpose was to gather information and write first drafts of the working papers for the loan loss review in which preliminary views as to the adequacy of the bad debt provisions were formed. Tom's view was that Eric should have gone further and expressed carefully considered opinions as to the provisions. Throughout the observations the participants sought to understand (and check) answers to calculations and results produced by models; making sense of and checking the input, answers and results contributed to the accuracy of the *workplace mathematics* done.

13.6.6 Learning

Learning was central to my participants' work and their competence. Understanding and evaluating situations/events and information from texts formed a key part of my participants' activities. Learning involved understanding information relating to the tasks in hand, including substantial amounts in financial form. Learning, including improving *workplace mathematics* skills, occurred:

- just through doing the job, i.e., through practice;
- through doing something new or relatively unfamiliar;
- throughout the review process; and
- through coaching on or near the job.

The learning observed is discussed and analysed in chapter 10. Skills involving *workplace mathematics* were learnt – see 2) of summary (section 10.8).

When I examined some incidents of learning that clearly involved *workplace mathematics*, I found that *workplace mathematics* itself was used as a tool to aid learning, with exploratory calculations being a frequently used distinctive technique – see chapter 10 generally and 1) and 3) of summary (section 10.8). This is a key finding.

13.6.7 Innovation

Past practice, resources immediately available in the work environment and professional practices mostly governed what was done. However, innovation was also a component of competent performance. Innovation was possible because participants were to a certain extent allowed to choose how to perform tasks in hand. In chapter

11, I set out incidents of innovation – most involving *workplace mathematics*. Most innovation observed involved the introduction of a new technique or idea into the work of accomplishing a task (rather than transfer of learning between or across tasks). As participants worked on tasks, they adapted artefacts or practices. They also imported techniques/ideas from outside the immediate work environment. The immediate sources of innovation were participants' experience and their own knowledge and skills.

13.6.8 Innovating in particular situations

I would like to suggest the way transfer is viewed within the situated cognition paradigm is recast. Perhaps transfer should be considered not from the perspective of transfer across situations but from the perspective that in certain circumstances new ideas/techniques are transferred into the work of performing a task in hand in order to enable the performance of the task. Such an approach allows the researcher to concentrate on reflective practice and innovation in practice. It places more emphasis on the person acting, and the educational/work history of that person. In appropriate circumstances it also puts emphasis on the power of collaboration. Additionally, it may free researchers to focus on both the **continuities** and discontinuities of practice across studies of everyday activities and work within the situated cognition and related traditions.

Closely related to the issue of innovation is the issue of the general versus the particular solution. My participants investigated and reasoned about particular events/situations, both real and hypothetical. The particular took precedence over the general. My participants generally used particular numbers, facts and circumstances to model particular situations/events, the notable exception being Eric's use of the Excel proforma for the loan summaries. However the particular was generally modelled using well theorized models/practices of commerce, accounting and auditing. This practice is not unlike that of Nunes et al.'s experienced workmen.

Now it is time to reunite performance with context.

13.7 Reconnecting with context; its importance

The distributed cognition framework places heavy emphasis on the contexts in which work is performed. In this study I recognise the importance of aspects of the contexts surrounding the work done and acknowledge their influence on what is done. I have not however made an extensive analysis of the organisation of resources and the tools (e.g., models, artefacts, processes and systems) used, as my focus was on my key questions relating to individuals – what is done and how it is done. Both are, however, a major determinant in what is done. In table 1 in subsection 12.3.5, I use a series of four diagrams to illustrate the setting and environment in which a task was realised. In addition to highlighting the participants' contributions, which are discussed extensively above, it demonstrates the importance of:

- the task in hand;
- the organisation of the audit, the audit team and teamwork;
- technology, professional practices, texts and information; and
- the environment in which the work was done.

Workplace mathematics is absent from this analysis; the analysis is too coarse grained to capture the *workplace mathematics* embedded in the work practice. Tasks in hand have been dealt with extensively in this thesis as they formed episodes for analysis. I now turn briefly to teamwork, the readily available resources and the immediate environment.

Hutchins' distributed cognition framework as applied to the team of navigators brings to fore the importance of team organisation and teamwork to effective task performance. The organisation of the audit team and its work enabled the process of auditing in a similar way. Tasks were allocated across the team taking account of their prior experience and knowledge, this being normal practice. The quality of briefing, the willingness of team members to seek guidance as tasks progressed and the openness of the review system also had an important bearing on the competence with which the task was accomplished, e.g., Ramesh clarified his uncertainties with Jerome before he assessed one of the vacant property provisions. Teamwork at its best depended upon good working relationships, e.g., Tom and Eric had a good working relationship leading to the situation where Tom taught Eric how to carry out analytical reviews of accounts while Eric's relationship with Cliff was far less productive. Prior to beginning a substantive task, participants spent time gathering together the information (from clients, team members and prior periods' papers) and papers which they considered necessary to make an adequate start on the task; this often included being told, or agreeing with other team members, how the task should be approached. Effective teamwork and discussion expanded the intellectual resources available to participants. This is an unsurprising but important finding. Tom set out clearly on several occasions how the assessment of the adequacy of bad provisions should be approached; this included a broad description of the mathematical/accounting models that should be used.

A further important finding is that, like Hutchins' team of navigators, my participants accomplished tasks through collaboration, not just the division of a task into a series of tasks. For example, Eric and Cliff jointly worked towards understanding the results produced by the Kookaburra debt valuation model and its commercial meaning; they shared the work and a new meaning emerged. Teamwork is not only a context in which *workplace mathematics* is realised, it is sometimes an essential part of performance.

Technology not only enabled *workplace mathematics,* it also helped to structure it. In composing texts, participants generally used calculators and PCs to perform all calculations other than the simplest. In other situations, electronic means of calculation were used as calculators (and PCs) were generally to hand, e.g., for exploratory calculations when trying to understand texts and situation. Computer packages, such as Word but particularly Excel, had an influence on both the method of calculation and the construction of models. Knowledge of accounting and auditing standards and practices and the related mathematical models also enabled and structured *workplace mathematics.* Customary well known models of accountancy were used if they were seen to be fit for purpose as were standard auditing models of presentation. Copies of client papers were modified so they could be used as audit working papers, and information and models were taken from client papers and records where they could be adapted with relatively little effort. Work done was structured by the previous year's working files, the audit plan and the generally accepted standards of accounting and auditing. Although access to technology and tools was generally enabling, it also had some limiting effects on what was done. For example, Joan used a standard goodwill valuation model to assess possible future cash flows that might be generated through the exploitation of the rights (goodwill) and know how acquired; this did not take account of the fact that the cost of the goodwill would not be tax deductible in the UK. Also sometimes the use of Word, as opposed to Excel, limited what could be presented easily within a table.

The past intellectual efforts of others were accessed through the customary models of accountancy and auditing and the use of client papers and the previous periods' papers. Technological tools used appropriately also allowed past intellectual efforts of others to be harnessed effectively as well as enabling accuracy and the saving of time and effort. All enabled the effectiveness of participants' own work increasing the complexity and sophistication of what was done as well as saving time and effort. This should not be lost even though I have focused on the *workplace mathematics* as realised by individuals in performance.

13.8 Conclusion

I set out to study how participants use and do mathematics in a non-specialist workplace. I expected to, and did, find my participants used mathematics to carry out their daily work and in the process developed a definition of *workplace mathematics*. That my participants used *workplace mathematics* extensively was not surprising. What was more surprising was the wide range of uses to which it was put. Generally my participants used *workplace mathematics* competently both when working alone and in conjunction with others. My findings as to the practice of *workplace mathematics* are described throughout this thesis; they are both extensive and complex. Even though they are situationally specific, there are strong similarities across participant, task and situation/event. Again this is not surprising because observations were made in the two assurance divisions of the same international accounting firm and most were directly related to the practice of auditing.

The mathematics used by my participants was not particularly sophisticated but was used in a sophisticated manner. Generally my participants used no more mathematics than is required to obtain a C grade at GCSE. The mathematics was performed competently – my participants made few significant lasting mistakes. However the manner in which it was used in *workplace mathematics* was often quite sophisticated, e.g., when Tom estimated the possible Kookaburra debt write off, and when Holly deduced that the payment of a dividend might be illegal. Even apparently straight forward practices such as the construction of the summary tables by Eric proved on inspection to contain much more mathematics than just the tabulation and totalling of balances. The sophistication observed derived in part from the ability to access past cognitive efforts of other which were locked into tools and technology, well theorized practices and models of auditing, accounting and commercial practice. It also derived from having:

- to collect and make sense of substantial amounts of information relating to the situations/events under investigation;
- to extract and connect facts, opinions and arguments relevant to the task in hand from that information; and
- to respond to interim findings and results as tasks unfolded and evolved.

In the situations reported in this thesis, usage of technology, practices and models and responding to results all involved extensive use of *workplace mathematics. Workplace mathematics* was also involved in the presentation and discussion of results. It is this diverse and complex usage of *workplace mathematics*, including specifically using it to enable the simplification of complexity, which gave rise to much of the sophistication, rather than the mathematics itself. On the other hand there are two respects in which the participants' work might be viewed as being pragmatic and not particularly sophisticated from a mathematical perspective; participants generally modelled the particular with which they were concerned and used iterative methods (particularly when they used the Excel programme) rather than starting from a general theoretical position.

Early analysis pointed up the importance of task and the participants' own knowledge and skills. Both remained important factors throughout the study with the task becoming the main unit for analysis. Focus upon the distribution of and nature of cognitive effort during task performance reinforced the centrality of participants' extant knowledge, skills and experience to effective performance. There are strong similarities between many of my findings and some of those reported in *Cognition in Practice, Street mathematics and school mathematics* and *Cognition in the Wild.* All this suggests that there is a prospect of developing a functional mathematics curriculum for use in secondary schools in the England that would be a sound foundation to enable competent *workplace mathematics* in everyday life and ordinary workplaces.

As I reflect upon the findings in this study and its theoretical foundations, two further themes of relevance to the development of a functional mathematics curriculum emerge:

- Following Wittgenstein and theories of practice, competence in mathematical practice requires teaching/coaching, learning and above all practice, with understanding, meaning and knowledge, and competence and expertise flowing from practice and reflection.
- The skilful exercise of mathematics in ordinary work is a matter primarily of practice and experience, with task, context, resources (including all forms of technology), and knowledge and skill shaping performance.

With these insights in mind, I now turn to consider the way forward including how the functional mathematics curriculum and further research could be influenced by this study. In particular in section 14.3, I discuss how my detailed findings can be used to underpin a functional mathematics curriculum.

14 The way forward

14.1 Introduction

Having summarized my findings at the end of the previous chapter, I now look at how this study can contribute to the development of an effective functional mathematics curriculum for secondary schools in the UK. First I draw attention to some limitations of the study that need to be borne in mind as I make suggestions about how my findings could be used. Finally, I end by considering possible avenues for future research to improve our knowledge of how mathematics is used in work and to help create a relevant functional mathematics curriculum.

Following the CBI definition of functional skills (CBI, 2006, p.4), by functional mathematics I mean skills in mathematics and the use of mathematics that provide an individual with the ability to tackle practical everyday tasks in the real world.

14.2 Limitations

My participants were highly qualified as compared with the general public and worked within a firm, where trainee accountants are articulate and numerate and now almost without exception graduates. Leonora, the administrative assistant, had 6 A*-C GCSEs, including a C in mathematics. The observations took place within two divisions of one large business; 70% of the observations took place during audits on client premises and 90+% while the participants worked on client affairs. The work observed did not require any mathematics beyond that studied for higher tier GCSE, except perhaps that involved in discounted cash flow analysis. The *workplace mathematics* observed included data display and presenting information, extensive financial modelling of events/situations and reasoning using numbers and the results from models. Nevertheless, limited practice was observed in many important areas. For example:

- Little routine use of statistics[1] was observed. My work relating to the building society suggests that statistics and related techniques can and do play a major role in monitoring and driving work and decisions throughout an organisation.
- No *workplace mathematics* practice involving the mathematics of space or space and time was observed.
- There were no observations of *workplace mathematics* in situations where most employees are required to follow certain processes and scripts (e.g., when applying for a mortgage and/or as in call centres). And
- There were no observations of the use of conventional forms of measurement.

Although the findings in this study are situationally specific, substantial continuities in practices were observed across participants and situations, and with other studies reported in three key texts (section 12.2). It is the existence of these continuities that justify making suggestions apposite to a functional mathematics curriculum.

[1] On the Bank audit, statistics were used extensively by a manager who performed a detailed analytical review of the Bank's draft accounts (not observed). Statistics are also used extensively to manager the accounting firm's business on a day to day basis. (Source: former senior partner.)

14.3 Implications of study for functional mathematics

In light of the limited range of *workplace mathematics* practice observed, only tentative suggestions can be made in relation to functional mathematics curricula. Nevertheless the study reveals much about the practice of arithmetic, spreadsheet algebra and mathematical reasoning in business today. Furthermore it was arguably an advantage to observe highly qualified and competent participants doing non-specialist work thoroughly as work done competently/expertly provides a sound guide to the nature of good practice and the necessary skills.

A key finding of this study is that competence/expertise depends upon two critical factors: there must be teaching/coaching/induction into practice[1] of mathematics and work incorporating mathematics; and individual understanding, knowledge and skill is only achieved through engaging in the practice[2] of work. Both competence and expertise in the use of *workplace mathematics* is dependent upon practice. On the other hand this does not mean that I am suggesting that practice alone is sufficient[3] to enable competence and/or expertise.

This study involves participants who were able to exercise considerable individual agency. This is important as it meant that they did much more than follow routine processes, thus giving insight to the skills necessary to identify and solve dilemmas/problems using *workplace mathematics*. The study also shows that extant knowledge and skills, and experience of individuals are key determinants of competence/expertise and that prior education and training as well as experience matters.

Thus a functional mathematics curriculum should enable students to become knowledgeable and skilful in the use of mathematics through extensive practice, with sophistication of use being developed as students work through the curriculum. My work suggests that the emphasis should be on knowing and knowing how through doing. Many of the skills requisite for proficiency cannot be acquired without performing realistic tasks or simulations thereof. My study also suggests that this presupposes spending time understanding the task, extracting and connecting information relevant to the task from information gathered from a variety of sources and performing the task itself. It involves choosing appropriate method/s to implement the task. It also presupposes making sure input and output is both accurate and complete; assessing answers for sense; interpreting them; and being prepared to revise answers in light of new information and new insights. It is to be noted here that traditional classroom word problems provide practice of mathematical procedures, but they do not simulate real life problems/dilemmas which require the use of mathematics to derive a good solution. If simulations of realistic tasks are to promote sophistication of use of relatively simple mathematics within functional mathematics then this implies creating substantial case studies. Ideally these would involve giving students superfluous (and possibly incomplete) information, in addition to creating problems for solution which both require identification and transformation, and sensible mathematically based solutions.

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1 Here practice refers to mathematical and work practices as used in school/education and workplaces. .

2 Here practice bears the meaning 'habitually perform', i.e., practising.

3 My study does not provide evidence that enables any comment to be made about sufficiency.

All the *workplace mathematics* observed in this study falls within the category of functional mathematics. My findings suggest that proficiency in functional mathematics depends, in part, upon three skill sets:

1) Using the appropriate mathematics proficiently (based upon findings in section 13.4, particularly subsections 13.4.1 and 13.4.3);

- being proficient at arithmetic, in particular knowing which operations to use and creating appropriate expressions,
- choosing effective and efficient methods of calculation. This implies using available electronic forms of calculation (or paper and pencil methods[1] if no electronic means are available) for all but the simplest calculations. This does not preclude the necessity for competence in being able to calculate without a calculator,
- using estimating and rounding practices appropriate to the situation,
- being proficient at mental calculation (where numbers are estimated or rounded to the appropriate degrees of significance if necessary),
- using spreadsheet algebra. This implies an underlying knowledge of writing general formulas and algebra, and
- using logic and reasoning as part of functional mathematics practice.

2) Using mathematics appropriately in context (based upon findings in subsections 13.4.2 and 13.4.4; section 13.5; and subsections 13.6.6 and 7);

- knowing how to use and using mathematics (including data displays, statistics and trend analyses) to simplify and organise information,
- knowing how to use, and using and creating some mathematical models. Also knowing when to use them. It probably means that, if appropriate, the particular rather than the general should be modelled using standard model frameworks that are known to be appropriate to the situation,
- knowing how to write texts incorporating mathematical information and how to use mathematics as warrant and rhetoric appropriately, and
- knowing how to interpret data and results derived from mathematical models.

3) Using mathematics socially to enable collaboration, communication and learning (based upon findings in section 13.5; subsection 13.4.1, particularly the paragraph headed ***Exploratory calculations;*** and subsections 13.6.5 to 8);

- being able to use numbers in conversation to describe and reason about events/situations,
- working together collaboratively as team performance matters,
- learning to use mathematics in or for work,
- using mathematics to learn – exploratory calculations being a key and important technique, and
- acquiring an open attitude to the need to learn and innovate.

[1] It is noteworthy that throughout my fieldwork I observed a paper and pencil method being used on one occasion only. I do not believe that my experience is necessarily representative of business practice.

This list is of relevance to someone working within the financial sector where that person is concerned with financial information and is able to exercise considerable individual agency. It is, of course, of greater or lesser relevance to others.

What this study shows is that a heavy burden is placed upon curriculum designers, if the desire is to develop proficiency in functional mathematics through the exploration of realistic tasks/situations. In particular, the practical exercises through which it is proposed that students should learn need to be designed with care and properly piloted. The choice of the tasks themselves will need to be chosen with care as their situational specificity will to a large extent determine both the mathematics used and the way in which it is used. It is also probable that student experience of the task and its realisation in the classroom may not achieve the initial expectations of the writer. Consequently all material should be piloted in the classroom and through, if necessary, examinations. Furthermore progression through a series of task is probably necessary to allow students to build up their competence gradually.

14.4 Lines of future enquiry

Further research is required both to find out more about how mathematics is used in work and to assist further in developing a relevant functional mathematics curriculum.

Much research has been done into the use of mathematics in work. Meta-analysis of existing studies would establish clearly what we know and indicate areas where further research is required. There may be a case for small scale qualitative studies into other areas, including those listed in section 14.1 above. The findings from the meta-analysis should be aimed partly at scoping out the mathematical skills and knowledge necessary to be competent in work and everyday life. The CBI report, *Working on the three Rs* (2006), has already started this process.

A second line of enquiry would to be to widen lines of enquiry into *workplace mathematics* by further considering:

- on a theoretical level, what constitutes competence/expertise in mathematics and *workplace mathematics,* and how we come to be competent/expert practitioners;
- the role of technology and resources – some work has already been done in this area by others including Hutchins (1995) and the team at the London Knowledge Laboratory, the Institute of Education, University of London (http://www.lkl.ac.uk);
- work in which A-level and degree level mathematics is used; and
- whether or not those in work would perform, or would have performed, more effectively and efficiently if they had a wider and deeper knowledge of and skills in aspects of mathematics.

A third line of enquiry would involve comparing existing and proposed curricula with good/effective practice in work and everyday life. This is particularly important because of the distorting effect of examinations and the implementation of other formal assessment systems on teaching and learning (Ofsted, 2006). I consider that this research should comprise more than a literature review. It should include observation of the functional mathematics curriculum as realised in student practice and so lead to action research projects that seek to improve teacher/student effectiveness via classroom practice, the curriculum and its assessment through using realistic tasks that incorporate *workplace mathematics*. The detailed case studies in chapters 5 and 7 to11 provide insights as to what the realistic tasks might encompass.

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Functional Mathematics Skills

Appendix 1

From the CBI Report *Working on the Three Rs* (2006)

7 Multiplication tables and mental arithmetic without using a calculator constitute an essential aid in all sorts of work activities.

8 The ability to interpret and respond to quantitative data is a key part of modern working life. Data of this type is presented not only to keep employees in the picture, but employees are also expected to interpret it sufficiently to contribute to problem solving and quality improvement.

9 Calculating and understanding percentages is a functional maths skill. Percentages are widely used in internal communications and in many jobs it is essential to be able to calculate them readily. A functionally numerate person should therefore both be able to calculate a percentage and interpret the significance of percentages communicated to them.

10 As well as percentages, a mathematically literate person will be able to work comfortably with fractions, decimals and ratios. For many organizations, the ability to use a formula is also highly desirable.

11 It is important for employees to have awareness of different measures and the ability to convert between them. Despite all the moves towards metrication, imperial and metric measures both remain in daily use. Employees need to be able to cope with that reality.

12 Spotting errors and rogue figures is an important element of functional maths. A functionally numerate employee will almost instinctively carry out a reality check and pause to check what may potentially be a rogue result.

13 Some basic understanding of odds and probabilities to enable people to make a more realistic assessment, rather than treating every risk as equally likely to happen, could form a useful element of functional mathematics.

14 Functional skills are skills that have a practical purpose. It is important to boost awareness of their potential application, particularly key elements of mathematical literacy, in real and different contexts.

Appendix 2

Client		Period
...... Bank plc		31 December 2003
Prepared by	**Date**	**W/P reference**
....	19 February 2008	

– credit update

Last reviewed	Half year 2003	
	Grade	**Provision, £m**
Grading and provision at last review	FG	-
New grade and provision	FG	-
Brief Description of company		
Conclusion of prior review		
Source		

Current facility position

	Facilities £'000	Outstandings £'000
x		
On balance sheet		
Off Balance sheet		
less Security		
Net at risk	-	-
Settlement facilities		-
Total	-	-

Update on credit position

How is the Bank going to get their money back?

Justification of provision

Conclusion

Manager Comments

Appendix 3

Client		Period
…… Bank plc		31 December 2003
Prepared by	**Date**	**W/P reference**
….. …..	19 February 2008	

– credit review

Date of latest credit application		
	Grade	**Provision, £m**
Grade and provision	FG	-
Brief description of company		
Source		

Current facility position

	Facilities £'000	Outstandings £'000
x		
On balance sheet		
Off Balance sheet		
less Security		
Net at risk	-	-
Settlement facilities		-
Total	-	-

Reason for original non performing status

Current key issues affecting status of credit

Financials

Reasons for concern/comfort

Conclusion

kpmg

Manager Comments

Appendix 4

Credit review approach

Key Issues:

- Security
- Cashflow
- Sale of assets
- Sale of the Bank's [redacted] debt

Financial Statement analysis

1. Gearing of company – can it sustain?
2. Evaluate net working capital (excluding the Bank's [redacted] debt) – how much cash can extract?
3. Fixed assets – valuation of fixed assets – how much do we need to discount for fire sale of assets?
4. Future cash flows – start at profit @ non-cash P&L items.

   If profitable but burning cash → can be OK as will make cash in the future – are profits going to transfer into cash in future?

   if making losses → is it due to non-cash items – they can continue as going concern if generating cash flow to fund working capital

   are large investments being made in a network → will be used in future and therefore generate cash flows. Has value to other companies as they will use network in the future

   if loss making but cash gen → need to evaluate business plan to see how company will return to profits as current situation is probably unsustainable

   if loss making and losing cash → business plan urgent to see how debt repaid. Company will need to increase revenues or decrease costs. Can cost reductions support revenues? If company focusing on revenue growth → look for external report to back up claims.

   Sale of subs/fixed assets → is sale possible in current market → what is likely price?

   Rights issue etc → how going to get rights issue away if company is in financial difficulty?

1.12.03.

Appendix 5

Wallaby

[redacted] Corporation – H1 financial statement review – 6 months to 30 September 2003

Gearing

Total debt of £673m (£20m due < 1 yr), results in gearing of 208% on equity of £323m.

Working capital

Net current assets are £819m. Stocks and debtors total £674m and there are cash balances of £772m. Based on this the total debt could be repaid immediately from working capital due to the high cash balances. The short term creditors are offset by the debtors and stock even if stock and debtors are written down by 80% the level of cash will be sufficient to meet the short term creditors and long term debt obligations.

Fixed assets

The company has £192m of fixed assets and £23m of investments. Therefore there are sufficient fixed asset balances to meet the debt repayments.

Profit and loss

Operating losses of £162m were made in the 6 month period. £91m of this related to the non-cash items of depreciation and amortisation. This resulted in a cash loss of £71m.

Cash Flow

Net cash inflows from operating activities were £65m, before exceptionals. Before working capital movements the total cash outflow was £21m. The net movement in working capital was £86m, the result of large decreases in stock and debtors. Stock decreased by 14% and debtors by 20%. Meanwhile the level of creditors increased. These working capital movements are not sustainable and therefore in future period we would expect the level of operating cash inflows to decrease unless there is an improvement in the level of cash generated profits.

Post exceptionals the operating cash outflow was £44m. The £109m of exceptionals can be broken down as follows: ESOP Settlement £35m; Restructuring costs £55m; and "Other" £19m. Net interest payments were £19m. Therefore net operating cash flows post interest but before exceptionals was £46m. This would result in the debt being repaid in 11 years (based on grossed up figure of £92m cash inflow), this assumes that the improvement in cash profits mitigates the current advantageous movements in working capital.

Based on the current improvements in operating performance we would expect that the level of cash generated profits will increase and therefore the above assumption will be met. Therefore the current level of debt can be paid off through operating cash flows. If this is not achieved the group has significant current assets that could be called upon to service the debt.